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MACRO SYNTHESIS OF WIDE AREA COMPUTER NETWORKS AT NONLINEAR COSTS

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Abstract. In practice, simple solutions of quick preliminary estimation of basic characteristics of a wide area computer network sometimes are needed. To achieve this goal, the backbone data transfer subnet and server set of computer networks are opportune to be examined. Considering the nonlinear continuous dependence of the costs of channels, routers, and servers on their performance and using the J.R. Jackson's partitioning theorem, an analytical model for these components of the network is defined. Based on this model, two optimization problems are formulated and examined: minimizing the average retention time of data processing requests and minimizing the summary cost of servers, channels and routers of the network. For both problems, numerical solution algorithms are proposed. These algorithms allow obtaining the expected solutions in continuous form, the respective performances being expressed by positive real numbers. Since the allowed performances of the mentioned entities are discrete, additional adjustments to the solutions in question, depending of the case, may be necessary. For such adjustments, two algorithms are proposed. One of them solves the problem by reducing it to that of the backpack. Another solves the problem in an heuristic manner based on the L. Kleinrock's resource concentration rule.

Keywords: *algorithm, channel, model, optimization, response time, router, server.*

Rezumat. În practică, uneori sunt necesare soluții simple de estimare preliminară rapidă a caracteristicilor de bază ale unei rețele de calculatoare de arie largă. În acest scop, este oportun de examinat subrețeaua magistrală de transfer date și setul de servere ale rețelelor de calculatoare. Considerând dependența neliniară continuă a costurilor canalelor, ruterelor și serverelor de performanța acestora și utilizând teorema de partiționare a lui J.R. Jackson, este definit un model analitic pentru aceste componente ale rețelei. Pe baza acestui model, sunt formulate și examinate două probleme de optimizare: minimizarea timpului mediu de reținere a solicitărilor de procesare a datelor și minimizarea costului sumar al serverelor, canalelor și ruterelor rețelei. Pentru ambele probleme, sunt propuși algoritmi de soluționare numerică. Acești algoritmi permit obținerea soluțiilor scontate în formă continuă, performanțele respective fiind exprimate prin numere reale pozitive. Deoarece performanțele permise ale entităților menționate sunt discrete, pot fi necesare, în funcție de caz, ajustări suplimentare ale soluțiilor în cauză. Pentru astfel de ajustări, sunt propuși doi algoritmi. Unul

dintre aceștia rezolvă problema reducând-o la cea a rucsacului. Celălalt rezolvă problema într-o manieră euristică, bazată pe utilizarea regulii lui L. Kleinrock de concentrare a resurselor.

Cuvinte cheie: *algoritm, canal, durată de răspuns, model, optimizare, ruter, server.*

1. Introduction

Because the creation and development of wide are computer networks involve considerable costs [1], minimizing these expenses is important. For this purpose many researches are carried out. In a general form, the problem of computer networks synthesis consists in determining of their topology, of the number, deployment, productivity, operating regimes and functions of servers and communication nodes, the capacity of data transfer channels, the distribution of user requests among servers and the ways of data transfer and routing in the network, which would ensure the extreme value of the used optimization criterion for the required quality of serving the user requests [2].

For large networks, the mathematical formalization and overall solution of the problem is very difficult. That is why various simplifications are usually resorted to, including the disaggregation of the general problem into several sub-problems [1,3,4].

Given the stochastic nature of computer network processes, in performance research, they are usually examined as queuing networks [4,5]. In the field of queuing networks, models and algorithms are developed, several analytical solutions are obtained. It is especially the case to mention the J.R. Jackson's partitioning theorem [6], the L. Kleinrock's square root law [7] and conservation law [5], the Gordon-Newell's theorem [8], the J. Buzen's algorithm [9], the BCMP networks [10], the MVA method [11], and the MVAC [12] and DAC [13] algorithms.

Search engines [14,15], Big data [16,17], and Cloud computing [18] have involved separate research with data centers [19,20], including their design, implementation, and performance analysis [21], their reliability [22], the placement of their virtual machines [23], and so on.

With refer to data networks, they are examined the necessary capacity of data channels at their linear [7] and power function [5] costs. Also, the needed capacity of computer network channels, routers and servers at linear dependence of their costs on their performance is examined in [24].

New steps in the development of computer networks are done by Grid computing [25, 26] and software defined networks (SDN) [27,28], including their design [29], the choosing of SDN operating system [30], the placement of controllers [31], etc.

However, the proposed methods and algorithms are, as a rule, relatively laborious, and sometimes in practice simple solutions are useful for the quick preliminary estimation of basic characteristics of a computer network. The purpose of this paper is to propose relatively simple algorithms for the determination of approximate necessary performances of channels, routers and servers of wide area computer networks at nonlinear costs of these components on their capacity.

2. A Simplified Model of Wide Area Computer Networks at Nonlinear Costs

Of course, the assumptions that the dependencies of costs of channels $C_{ci}(d_{ci})$ $(i = \overline{1, k_c})$, of routers $C_{ri}(\mu_{ri})$ $(i = \overline{1, k_r})$, and of servers $C_{si}(\mu_{si})$ $(i = \overline{1, k_s})$ on their capacities are linear, used in [24], allow only very approximate solutions regarding the required capacities $d_{ci}(i =$

$\overline{1,k_c}$) of channels, that μ_{ri}($i = \overline{1,k_r}$) of routers, and that μ_{si}($i = \overline{1,k_s}$) of servers. Here k_c, k_r, and k_s are the number, respectively, of channels, of routers, and of severs in the network.

Some improvements in the accuracy of the [24] solutions can be obtained by using the $C_{ci}(d_{ci})$ ($i = \overline{1,k_c}$), $C_{ri}(\mu_{ri})$ ($i = \overline{1,k_r}$), and $C_{si}(\mu_{si})$ ($i = \overline{1,k_s}$) continuous nonlinear dependencies of reasonable increasing behavior (without first or second order discontinuities, i.e. the first order derivatives are continuous functions). The assumption that dependencies $C_{ci}(d_{ci})$ ($i = \overline{1,k_c}$), $C_{ri}(\mu_{ri})$ ($i = \overline{1,k_r}$), and $C_{si}(\mu_{si})$ ($i = \overline{1,k_s}$) are increasing is natural, because increasing the entity's capacity leads to reducing the duration of user requests retention. Thus, if there are such cases (equal or higher cost at lower performance) for some alternatives of constituting the respective network components, these are eliminated (not taken into account), without reducing the chances of obtaining the optimal solution.

So, from the multitude of computer network macro synthesis aspects, the model includes those related to estimating the necessary capacities of their servers, data transfer channels or trunks (hereafter channels or DTT), and routers at their nonlinear costs on their performance. The model is an improvement of the one proposed and researched in [24].

Because computer networks operation has a stochastic character, they usually are examined as networks of queues. Notes [24]:

- rate β of the summary flow of requests to servers;
- rate γ of the summary flow of packets generated by user requests and the responses to them;
- the mean number g of packets per request, $g = \gamma/\beta$;
- packet rate through channel i - λ_{ci} and router j - λ_{rj};
- rate λ_{sl} of requests to server l;
- average size V_i of a packet transmitted through channel i;
- the average retention of a packet at channel i - T_{ci} and at router j - T_{rj};
- the average retention of a request at the server l - T_{sl} and in the network - T;
- summary cost of channels - C_c, of routers − C_r, of servers − C_s, and of the network - C.

Moreover, as in [24], it is considered that flows λ_{ci} ($i = \overline{1,k_c}$), λ_{rj} ($j = \overline{1,k_r}$), and λ_{sl} ($l = \overline{1,k_s}$) are elementary and the time of packet transmission by channels, of packet processing by routers, and of request processing by servers are exponentially distributed. At the same time, unlike in [24], the dependences $C_{ci}(d_{ci})$ ($i = \overline{1,k_c}$), $C_{ri}(\mu_{ri})$ ($i = \overline{1,k_r}$), and $C_{si}(\mu_{si})$ ($i = \overline{1,k_s}$) are continuous increasing of reasonable behavior.

It will be examined the steady-state operation of the network, which exists if the relationships occur:

$$\mu_{ci} > \lambda_{ci},\ i = \overline{1,k_c};\ \mu_{rj} > \lambda_{rj},\ j = \overline{1,k_r};\ \mu_{sl} > \lambda_{sl},\ l = \overline{1,k_s}, \tag{1}$$

where $\mu_{ci} = d_{ci}/V_i$ is the packet transmission rate by channel i, $i = \overline{1,k_c}$.

Under the conditions and assumptions defined above, a computer network can be examined as an open exponential network of queues with a homogeneous flow of requests operating in steady-state.

According to the Jackson's partitioning theorem [6], any open exponential network of queues can be examined as a set of isolated exponential queuing systems. Then the average time T_i of a request in the queuing system i of the network is determined [4] according to the relation $T_i = 1/(\mu_i - \lambda_i)$, where λ_i is the request flow rate, and μ_i − the request serving rate by station i. So, the relations take place:

$$T_{ci} = \frac{V_i}{d_{ci} - \lambda_{ci} V_i},\ i = \overline{1,k_c}; \tag{2}$$

$$T_{ri} = \frac{1}{\mu_{ri} - \lambda_{ri}}, i = \overline{1, k_r}; \quad (3)$$

$$T_{si} = \frac{1}{\mu_{si} - \lambda_{si}}, i = \overline{1, k_s}. \quad (4)$$

Taking into account Eq. (2)-(4), is obtained

$$T = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\lambda_{ci}T_{ci} + \sum_{i=1}^{k_r}\lambda_{ri}T_{ri}\right) + \frac{1}{\beta}\sum_{i=1}^{k_s}\lambda_{si}T_{si} = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci} - \lambda_{ci}V_i} + \sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri} - \lambda_{ri}} + g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si} - \lambda_{si}}\right). \quad (5)$$

Also, evidently, it takes place

$$C = C_c + C_r + C_s = \sum_{i=1}^{k_c}C_{ci}(d_{ci}) + \sum_{i=1}^{k_r}C_{ri}(\mu_{ri}) + \sum_{i=1}^{k_s}C_{si}(\mu_{si}). \quad (6)$$

The queuing network model is determined by Eq. (5), Eq. (6) and conditions of InEq. (1). With this model, two problems of determining the performance required for common-use components of computer networks are usually of interest:

- minimizing the average retention time T in the network of the requests of data processing;
- minimizing the summary cost C of servers, channels and routers of the computer network.

These are explored in Sections 3 and 4, respectively.

3. Minimizing the Average Response Time to User Requests

Problem 1 - minimizing the average response time to user requests at nonlinear costs with common-use components of the wide area computer network. Let, for the model {Eq. (5), Eq. (6)} and conditions of InEq. (1), the parameters are known: C^*; β; γ; k_c; k_r; k_s; λ_{ci}, V_i, $C_{ci}(d_{ci})$ ($i = \overline{1, k_c}$); λ_{ri}, $C_{ri}(\mu_{ri})$ ($i = \overline{1, k_r}$); λ_{si}, $C_{si}(\mu_{si})$ ($i = \overline{1, k_s}$), where C^* is the maximum allowed total cost of servers, channels, and routers of the network. It is required to determine the necessary performances d_i ($i = \overline{1, k_c}$) of channels, those μ_{ri} ($i = \overline{1, k_r}$) of routers and those μ_{si} ($i = \overline{1, k_s}$) of network servers that would minimize the average retention time T of data processing requests at the total cost C of servers, channels and routers which would not exceed the given value C^*, i.e.:

$$T = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci} - \lambda_{ci}V_i} + \sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri} - \lambda_{ri}} + g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si} - \lambda_{si}}\right) \to \min \quad (7)$$

upon compliance with the restriction

$$C = C_c + C_r + C_s = \sum_{i=1}^{k_c}C_{ci}(d_{ci}) + \sum_{i=1}^{k_r}C_{ri}(\mu_{ri}) + \sum_{i=1}^{k_s}C_{si}(\mu_{si}) \le C^* \quad (8)$$

in conditions of InEq. (1).

Solving. From the essence of the problem, it is obvious that the solution of the problem corresponds to the limit value in Eq. (8), that is, $C = C^*$. Then the problem {Eq. (7), Eq. (8)} can be solved using the method of Lagrange multipliers. The respective Lagrangian L is

$$L = \frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)+ + \chi(\sum_{i=1}^{k_c}C_{ci}(d_{ci})+\sum_{i=1}^{k_r}C_{ri}(\mu_{ri})+\sum_{i=1}^{k_s}C_{si}(\mu_{si})-C^*), \tag{9}$$

the conditioned optimization problem {Eq. (7), Eq. (8)} reducing to an unconditioned optimization problem – minimizing *L*. Here χ is the Lagrange multiplier.

The Lagrangian *L* contains $k_c + k_r + k_s$ +1 unknowns: χ; d_{ci} ($i = \overline{1, k_c}$), μ_{ri} ($i = \overline{1, k_r}$), and μ_{si} ($i = \overline{1, k_s}$). To minimize *L*, the partial derivatives of *L* with respect to the unknowns are determined; they are equalized to 0, resulting a system of $k_c + k_r + k_s$ +1 equations. By solving this system, the analytical expressions for the performances d_{ci} ($i = \overline{1, k_c}$), μ_{ri} ($i = \overline{1, k_r}$), and μ_{si} ($i = \overline{1, k_s}$) can be obtained. One has:

$$\begin{cases} \dfrac{\partial L}{\partial d_{ci}} = \dfrac{1}{\gamma}\cdot\dfrac{-\lambda_{ci}V_i}{(d_{ci}-\lambda_{ci}V_i)^2}+\chi\dfrac{\partial C_{ci}(d_{ci})}{\partial d_{ci}} = 0,\ i=\overline{1,k_c} \\ \dfrac{\partial L}{\partial \mu_{ri}} = \dfrac{1}{\gamma}\cdot\dfrac{-\lambda_{ri}}{(\mu_{ri}-\lambda_{ri})^2}+\chi\dfrac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}c_{ri} = 0,\ i=\overline{1,k_r} \\ \dfrac{\partial L}{\partial \mu_{si}} = \dfrac{g}{\gamma}\cdot\dfrac{-\lambda_{si}}{(\mu_{si}-\lambda_{si})^2}+\chi\dfrac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}} = 0,\ i=\overline{1,k_s} \\ \dfrac{\partial L}{\partial \chi} = \sum_{i=1}^{k_c}C_{ci}(d_{ci})+\sum_{i=1}^{k_r}C_{ri}(\mu_{ri})+\sum_{i=1}^{k_s}C_{si}(\mu_{si})-C^* = 0. \end{cases} \tag{10}$$

From the system of Eqs. (10), provided that the restrictions on InEq. (1) are satisfied, it follows that the equalities must hold:

$$\chi\gamma(d_{ci}-\lambda_{ci}V_i)^2\frac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}-\lambda_{ci}V_i = 0,\ i=\overline{1,k_c}; \tag{11}$$

$$\chi\gamma(\mu_{ri}-\lambda_{ri})^2\frac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}-\lambda_{ri} = 0,\ i=\overline{1,k_r}; \tag{12}$$

$$\chi\gamma(\mu_{si}-\lambda_{si})^2\frac{\gamma}{g}\cdot\frac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}-\lambda_{si} = 0,\ i=\overline{1,k_s}; \tag{13}$$

$$\sum_{i=1}^{k_c}C_{ci}(d_{ci})+\sum_{i=1}^{k_r}C_{ri}(\mu_{ri})+\sum_{i=1}^{k_s}C_{si}(\mu_{si})-C^* = 0. \tag{14}$$

In general case, obtaining, based on Eqs. (11)-(14), the analytical expression for χ, and then for the expected capacities d_{ci} ($i = \overline{1, k_c}$), μ_{ri} ($i = \overline{1, k_r}$), and μ_{si} ($i = \overline{1, k_s}$) is not possible. But their values can be obtained, under certain conditions, using a numerical method for solving equations, for example the dichotomy method.

According to Eqs. (11)-(14) and taking into account the conditions of InEq. (1) and the positive increasing character (by definition) of functions $C_{ci}(d_{ci})$ ($i = \overline{1, k_c}$), $C_{ri}(\mu_{ri})$ ($i = \overline{1, k_r}$), and $C_{si}(\mu_{si})$ ($i = \overline{1, k_s}$), the first factor of the sum of the left-hand side of Eqs. (11)-(14) at $\chi > 0$ is positive. So, the optimal solution is obtained at $\chi > 0$.

From Eqs. (11)-(14), as a result of some simple transformations, one obtains:

$$d_{ci} = \lambda_{ci} V_i + \sqrt{\frac{\lambda_{ci} V_i}{\chi\gamma \dfrac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}}},\ i = \overline{1, k_c}; \tag{15}$$

$$\mu_{ri} = \lambda_{ri} + \sqrt{\frac{\lambda_{ri}}{\chi\gamma \dfrac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}}},\ i = \overline{1, k_r}; \tag{16}$$

$$\mu_{si} = \lambda_{si} + \sqrt{\frac{g\lambda_{si}}{\chi\gamma \dfrac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}}},\ i = \overline{1, k_s}. \tag{17}$$

The numerical solution of the system of Eqs. (14)-(17) is considerably simplified if all the products $\chi \dfrac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}$ $(i = \overline{1, k_c})$, $\chi \dfrac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}$ $(i = \overline{1, k_r})$, and $\chi \dfrac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}$ $(i = \overline{1, k_s})$ are increasing with respect to χ or all these products are decreasing with respect to χ.

Indeed (***Scenario 1***), from Eqs. (15)-(17) it follows that if all the mentioned products are increasing with respect to χ, then the functions $d_{ci}(\chi)$ $(i = \overline{1, k_c})$, $\mu_{ri}(\chi)$ $(i = \overline{1, k_r})$, and $\mu_{si}(\chi)$ $(i = \overline{1, k_s})$ are decreasing (such a case occurs, for example, for the linearly increasing functions $C_{ci}(d_{ci})$ $(i = \overline{1, k_c})$, $C_{ri}(\mu_{ri})$ $(i = \overline{1, k_r})$, and $C_{si}(\mu_{si})$ $(i = \overline{1, k_s})$ – the case examined in [24]). Under such conditions and taking into account that all functions $C_{ci}(d_{ci})$ $(i = \overline{1, k_c})$, $C_{ri}(\mu_{ri})$ $(i = \overline{1, k_r})$, and $C_{si}(\mu_{si})$ $(i = \overline{1, k_s})$ are increasing by definition, it follows that all functions $C_{ci}(\chi)$ $\left(i = \overline{1, k_c}\right)$, $C_{ri}(\chi)$ $(i = \overline{1, k_r})$, and $C_{si}(\chi)$ $(i = \overline{1, k_s})$ are decreasing. So, the function $C(\chi) = \sum_{i=1}^{k_c} C_{ci}(\chi) + \sum_{i=1}^{k_r} C_{ri}(\chi) + \sum_{i=1}^{k_s} C_{si}(\chi)$ is also decreasing. Namely, such a case usually occurs in practice.

At the same time, on the contrary (***Scenario 2***), if all the mentioned above products are decreasing with respect to χ, then the functions $d_{ci}(\chi)$ $(i = \overline{1, k_c})$, $\mu_{ri}(\chi)$ $(i = \overline{1, k_r})$, and $\mu_{si}(\chi)$ $(i = \overline{1, k_s})$ are increasing. Respectively, all functions $C_{ci}(\chi)$ $\left(i = \overline{1, k_c}\right)$, $C_{ri}(\chi)$ $(i = \overline{1, k_r})$, $C_{si}(\chi)$ $(i = \overline{1, k_s})$, and $C(\chi)$ are increasing. Such a case is unlikely to occur in practice, but anyway it can be taken into account relatively simply.

Scenario 3, in which the numerical solution of the system of Eqs. (14)-(17) is relatively simple, consists of the following. Each of the mentioned above products is increasing or decreasing and, at the same time, the function $C(\chi)$ is increasing or decreasing with respect to χ.

It can be seen that Scenarios 1 and 2 are particular cases of Scenario 3. Therefore, only Scenario 3 can be examined. Let the quantities ε, ε_c, ε_r, and ε_s be the absolute values of the accuracy of determining the optimal values of unknowns C, d_{ci} $(i = \overline{1, k_c})$, μ_{ri} $(i = \overline{1, k_r})$, and μ_{si} $(i = \overline{1, k_s})$, respectively, and $a > 1$ and b are constants used to determine the character of the dependence $C(\chi)$ and the interval of values [χ; χ+], which contains the value χ^* corresponding to the optimal solution. Then **Algorithm 1** for numerical solving the system of Eqs. (14)-(17) for Scenario 3 using the dichotomy method consists of the following.

1. Initial data: C^*; β; γ; k_c; k_r; k_s; a; ε; ε_c; ε_r; ε_s; λ_{ci}, V_i, $C_{ci}(d_{ci})$ $(i = \overline{1, k_c})$; λ_{ri}, $C_{ri}(\mu_{ri})$ $(i = \overline{1, k_r})$; λ_{si}, $C_{si}(\mu_{si})$ $(i = \overline{1, k_s})$.
2. Determine the character of the dependence $C(\chi)$. $\chi := 1$, $b := 1$.

2.1. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on the solution of Eqs. (15)-(17), taking into account the values of the quantities ε_c, ε_r, and ε_s.

2.2. $C_{ci}(\chi) := C_{ci}(d_{ci}(\chi)), i = \overline{1, k_c}$; $C_{ri}(\chi) := C_{ri}(\mu_{ri}(\chi)), i = \overline{1, k_r}$; $C_{si}(\chi) := C_{si}(\mu_{si}(\chi)), i = \overline{1, k_s}$.

2.3. $C(\chi) := \sum_{i=1}^{k_c} C_{ci}(\chi) + \sum_{i=1}^{k_r} C_{ri}(\chi) + \sum_{i=1}^{k_s} C_{si}(\chi)$. $\Delta C := C(\chi) - C^*$. If $|\Delta C| < \varepsilon$, then go to Step 7.

2.4. If $b = 1$, then $C_o(\chi) := C(\chi)$, $\chi := a\chi$ and go to Step 2.1.

2.5. If $C(\chi) < C_o(\chi)$, then the function $C(\chi)$ is decreasing and go to Step 5.

3. Determine the interval $[\chi^-; \chi^+]$ for the increasing function $C(\chi)$.

3.1. If $C_o(\chi) < C^*$, then $\chi := 1$ and go to Step 3.8.

3.2. $\chi^+ := 1$, $\chi := \chi/a$.

3.3. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on the solution of Eqs. (15)-(17), taking into account the values of the quantities ε_c, ε_r, and ε_s.

3.4. $C_{ci}(\chi) := C_{ci}(d_{ci}(\chi)), i = \overline{1, k_c}$; $C_{ri}(\chi) := C_{ri}(\mu_{ri}(\chi)), i = \overline{1, k_r}$; $C_{si}(\chi) := C_{si}(\mu_{si}(\chi)), i = \overline{1, k_s}$.

3.5. $C(\chi) := \sum_{i=1}^{k_c} C_{ci}(\chi) + \sum_{i=1}^{k_r} C_{ri}(\chi) + \sum_{i=1}^{k_s} C_{si}(\chi)$. $\Delta C := C(\chi) - C^*$. If $|\Delta C| < \varepsilon$, then go to Step 7.

3.6. If $\Delta C > 0$, then $\chi^+ := \chi$, $\chi := \chi/a$ and go to Step 3.3.

3.7. $\chi^- := \chi$, the interval $[\chi^-; \chi^+]$ is determined and go to Step 4.

3.8. If $C(\chi) > C^*$, then $\chi+ := \chi$, the interval $[\chi-; \chi+]$ is determined and go to Step 4.

3.9. $\chi^- := \chi$, $\chi := a\chi$.

3.10. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on the solution of Eqs. (15)-(17), taking into account the values of the quantities ε_c, ε_r, and ε_s.

3.11. $C_{ci}(\chi) := C_{ci}(d_{ci}(\chi)), i = \overline{1, k_c}$; $C_{ri}(\chi) := C_{ri}(\mu_{ri}(\chi)), i = \overline{1, k_r}$; $C_{si}(\chi) := C_{si}(\mu_{si}(\chi)), i = \overline{1, k_s}$.

3.12. $C(\chi) := \sum_{i=1}^{k_c} C_{ci}(\chi) + \sum_{i=1}^{k_r} C_{ri}(\chi) + \sum_{i=1}^{k_s} C_{si}(\chi)$. $\Delta C := C(\chi) - C^*$. If $|\Delta C| < \varepsilon$, then go to Step 7.

3.13. If $\Delta C < 0$, then go to Step 3.9.

3.14. $\chi^+ := \chi$, the interval $[\chi^-; \chi^+]$ is determined.

4. Determine the solution, knowing the interval $[\chi^-; \chi^+]$ and taking into account that the function $C(\chi)$ is increasing.

4.1. $\chi := (\chi^- + \chi^+)/2$.

4.2. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on solving the Eqs. (15)-(17), taking into account the values of the quantities ε_c, ε_r, and ε_s.

4.3. $C_{ci}(\chi) := C_{ci}(d_{ci}(\chi)), i = \overline{1, k_c}$; $C_{ri}(\chi) := C_{ri}(\mu_{ri}(\chi)), i = \overline{1, k_r}$, and $C_{si}(\chi) := C_{si}(\mu_{si}(\chi)), i = \overline{1, k_s}$.

4.4. $C(\chi) := \sum_{i=1}^{k_c} C_{ci}(\chi) + \sum_{i=1}^{k_r} C_{ri}(\chi) + \sum_{i=1}^{k_s} C_{si}(\chi)$. $\Delta C := C(\chi) - C^*$. If $|\Delta C| < \varepsilon$, then go to Step 7.

4.5. If $\Delta C > 0$, then $\chi^+ := \chi$, otherwise $\chi^- := \chi$ and go to Step 5.1.

5. Determine the interval $[\chi^-; \chi^+]$ similarly to Step 3, but taking into account that the function $C(\chi)$ is decreasing.

6. Determine the solution, knowing the interval $[\chi^-; \chi^+]$, similarly to Step 4, but taking into account that the function $C(\chi)$ is decreasing.

7. Determine T^* according to (7). The solution is obtained.

Some comments on Algorithm 1. Steps 3.8-3.14 determine χ^+, given that Step 3.1 determined the initial value of χ^-. Conversely, Steps 3.3-3.7 determine χ^-, given that Step 3.2 determined the initial value of χ^+. It should also be noted that the values of the quantities ε_c, ε_r, and ε_s must be sufficiently small with respect to that of the quantity ε for Algorithm 1 to converge to the optimal solution.

Remark. Of course, the values of quantities μ_{si} ($i = \overline{1, k_s}$), d_{ci} ($i = \overline{1, k_c}$), and μ_{ri} ($i = \overline{1, k_r}$), obtained according to Algorithm 1, are positive real numbers, and the capacities of the channels, routers and servers that could be used in the network belong to a finite set. Therefore, an additional adaptation of the obtained solution is necessary.

Adjusting the solution of Problem 1, so that the allowed capacities of channels, routers, and servers be used, is performed in a similar manner to that described in Algorithms 1 and 2 for adjusting the solution to Problem 1 in [24] with the necessary adaptations. These are described below.

From the essence of the problem it follows that the value of the average retention time T^* of data processing requests is decreasing with respect to the maximum allowed total cost C^* of channels, routers, and servers. So, the smaller the value of the difference $\Delta C = C^* - C^{**}$, the smaller the value of the difference $\Delta T = T^{**} - T^*$, where C^{**} and T^{**} are the values of quantities C and T at allowed values of the capacities of channels, routers, and servers of the network. From this also results the essence of the problem of the necessary adjustment of the solution of Problem 1 mentioned in the remark above.

Problem 2 – adjustment of the solution of Problem 1. Let the initial data of Problem 1 and, also, the increasing sequences of the capacities of the components that can be used in the network are known: D_{ci} ($i = \overline{1, n_c}$) of the data transfer rates of the channels; D_{ri}($i = \overline{1, n_r}$) of the packet processing rates by the routers, and D_{si}($i = \overline{1, n_s}$) of the request processing rates by the servers. It is required to adjust the solution to the problem {Eq. (7), Eq. (8)} obtained according to Algorithm 1 so that, at allowed values of the capacities of channels, routers and servers of the network and respecting the conditions of InEqs. (1), the value $\Delta C = C^* - C^{**}$ is minimized.

Solution by reduction to the backpack problem. Obviously, the adjusted solution (allowable capacities of the basic components) should be as close as possible to the original one. Therefore, it is unlikely that it would be appropriate to increase the capacity of a component by more than the next value in the range of allowed capacities.

This aspect is the basis of **Algorithm 2** for the heuristic solution of Problem 2:

1. Initial data: C^*, T^*, θ, γ; D_{ci}, $i = \overline{1, n_c}$; D_{ri}, $i = \overline{1, n_r}$; D_{si}, $i = \overline{1, n_s}$; $C_{ci}(d_{ci})$, λ_{ci}, V_i, d_i, $i = \overline{1, k_c}$; $C_{ri}(\mu_{ri})$, λ_{ri}, μ_{ri}, $i = \overline{1, k_r}$; $C_{si}(\mu_{si})$, λ_{si}, μ_{si}, $i = \overline{1, k_s}$.
2. $h \coloneqq \max\{D_{ci}, i = \overline{1, n_c};\ D_{ri}, i = \overline{1, n_r};\ D_{si}, i = \overline{1, n_s}\} + 1.$
3. $D_{c,n_c+1} \coloneqq h$; $D_{r,n_r+1} \coloneqq h$; $D_{s,n_s+1} \coloneqq h$.
4. Taking into account the conditions of InEqs. (1), one determines:
$$u_{ci} := z | \min_{z=\overline{1,n_{c+1}}} \{D_{cz} \geq \lambda_{ci}\}, i = \overline{1, k_c};$$
$$u_{ri} := z | \min_{z=\overline{1,n_r+1}} \{D_{rz} \geq \lambda_{ri}\}, i = \overline{1, k_r};$$
$$u_{si} := z | \min_{z=\overline{1,n_{s+1}}} \{D_{sz} \geq \lambda_{si}\}, i = \overline{1, k_s}.$$
5. $u_c := \max_{i=\overline{1,k_c}} \{u_{ci}\}, i = \overline{1, k_c}$; $u_r := \max_{i=\overline{1,k_r}} \{u_{ri}\}, i = \overline{1, k_r}$; $u_s := \max_{i=\overline{1,k_s}} \{u_{si}\}, i = \overline{1, k_s}$;
6. If $u_c = n_c + 1$ or $u_r = n_r + 1$ or $u_s = n_s + 1$, then the problem has no solutions, because the conditions of InEqs. (1) for the operation of the network in a steady-state are not met.
7. Determine the allowed capacities of the basic components according to the solution to Problem 1:

$$\check{d}_i := D_{cz} | \{D_{cz} \le d_{ci} < D_{c,z+1}\}, z_{ci} := z, i = \overline{1, k_c};$$
$$\check{\mu}_{ri} := D_{rz} | \{D_{rz} \le \mu_{ri} < D_{r,z+1}\}, z_{ri} := z, i = \overline{1, k_r};$$
$$\check{\mu}_{si} := D_{sz} | \{D_{sz} \le \mu_{si} < D_{s,z+1}\}, z_{si} := z, i = \overline{1, k_s}.$$

8. The minimum reasonable capacities of the basic entities are determined:

$$\bar{d}_{ci} := \max\{D_{cu_{ci}}; \check{d}_{ci}\}, i = \overline{1, k_c};$$
$$\bar{\mu}_{ri} := \max\{D_{ru_{ri}}; \check{\mu}_{ri}\}, i = \overline{1, k_r};$$
$$\bar{\mu}_{si} := \max\{D_{su_{si}}; \check{\mu}_{si}\}, i = \overline{1, k_s}.$$

9. Determine the summary costs at the minimum reasonable capacities of the basic entities

$$\overline{C} = \sum_{i=1}^{k_c} C_{ci}(\overline{d}_{ci}) + \sum_{i=1}^{k_r} C_{ri}(\overline{\mu}_{ri}) + \sum_{i=1}^{k_s} C_{si}(\overline{\mu}_{si}).$$

10. The reserve of available financial resources is determined

$$\Delta\bar{C} := C^* - \bar{C}.$$

11. For each basic network component i, the additional cost of replacing its current capacity with the next one from the allowed range of capacities is determined:
   a) if $\bar{d}_{ci} \le d_{ci}$, then $\Delta C_{ci} := C_{ci}(D_{c,z_{ci}+1}) - C_{ci}(D_{cz_{ci}})$, otherwise $\Delta C_{ci} := 0$, $i = \overline{1, k_c}$;
   b) if $\bar{\mu}_{ri} \le \mu_{ri}$, then $\Delta C_{ri} := C_{ri}(D_{r,z_{ri}+1}) - C_{ri}(D_{rz_{ri}})$, otherwise $\Delta C_{ri} := 0$, $i = \overline{1, k_r}$;
   c) if $\bar{\mu}_{si} \le \mu_{si}$, then $\Delta C_{si} := C_{si}(D_{s,z_{si}+1}) - C_{si}(D_{sz_{si}})$, otherwise $\Delta C_{si} := 0$, $i = \overline{1, k_s}$.
12. An algorithm is applied to solve the backpack problem: among the quantities ΔC_{ci} ($i = \overline{1, k_c}$), ΔC_{ri} ($i = \overline{1, k_r}$), and ΔC_{si} ($i = \overline{1, k_s}$) to select those that would minimize $\Delta C = C^* - C^{**}$.

Solution by resource concentration. Evidently, if $C^{**} \ne C^*$, then the duration T should be calculated according to expression (5). Therefore, in such cases, taking into account the opportunity of resource concentration [5], when increasing the capacity it is appropriate to base on the rule: component i is preferable to component j, if $\Delta C_i/\Delta T(i) < \Delta C_j/\Delta T(j)$, where $\Delta T(i)$ is the decrease in duration T caused by the increase ΔC_i of the cost of component j. Following this rule is all the more successful, the larger the values of the quantities n_c, n_r, and n_s.

This rule is the basis of **Algorithm 3** for the heuristic solution of Problem 2:

1-11. Steps 1-11 coincide with Steps 1-11 of Algorithm 2.

12. $g_{ci} := \Delta C_{ci}/\Delta T(c,i)$, $i = \overline{1, k_c}$; $g_{ri} := \Delta C_{ri}/\Delta T(r,i)$, $i = \overline{1, k_r}$; $g_{si} := \Delta C_{si}/\Delta T(s,i)$, $i = \overline{1, k_s}$.
13. The arrangement of quantities g_{ci} ($i = \overline{1, k_c}$), g_{ri} ($i = \overline{1, k_r}$), and g_{si} ($i = \overline{1, k_s}$) in ascending order forming two arrays: a_i, b_i, $i = \overline{1, k_c + k_r + k_s}$, where a_i specifies the type (c, r or s), and b_i – the ordinal number of the capacity of the basic network component in the respective capacity range ($\overline{1, k_c}$, $\overline{1, k_r}$ or $\overline{1, k_s}$).
14. Increase to the next capacity in the capacity sequence in ascending order of b_i up to $b_i | \min\{\Delta C_i \geqslant 0\}$. The adjusted heuristic solution is obtained.

Problem 3 - minimizing the total cost of servers, channels, and routers of wide area computer networks at nonlinear costs. Let, for the model {Eq. (5), Eq. (6)} and conditions of InEqs (1), the characteristics are known: T^*; θ; γ; k_c; k_r; k_s; λ_{ci}, V_i, $C_{ci}(d_{ci})$, $i = \overline{1, k_c}$; λ_{ri}, $C_{ri}(\mu_{ri})$, $i = \overline{1, k_r}$; λ_{si}, $C_{si}(\mu_{si})$, $i = \overline{1, k_s}$. Here T^* is the maximum allowed average retention time in the network of data processing requests. It is required to determine the necessary capacities μ_{si} ($i = \overline{1, k_s}$) of servers, those d_{ci} ($i = \overline{1, k_c}$) of channels, and those μ_{ri} ($i = \overline{1, k_r}$) of the network routers that would ensure a minimum total cost C of servers, channels, and routers at the average retention time T in the network of data processing requests that would not exceed the given value T^*, i.e.

$$C = C_c + C_r + C_s = \sum_{i=1}^{k_c} C_{ci}(d_{ci}) + \sum_{i=1}^{k_r} C_{ri}(\mu_{ri}) + \sum_{i=1}^{k_s} C_{si}(\mu_{si}) \to \min \tag{18}$$

upon compliance with the restriction

$$T=\frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)\le T^*. \tag{19}$$

in conditions of InEqs (1).

Solving. From the essence of the problem, it is obvious that the solution corresponds to the limit value in Eq. (19), that is, $T = T^*$. Then the problem {Eq. (18), Eq. (19)} in conditions of InEqs (1) can be solved using the method of Lagrange multipliers. The respective Lagrangian L is

$$\begin{aligned}L=&\sum_{i=1}^{k_c}C_{ci}(d_{ci})+\sum_{i=1}^{k_r}C_{ri}(\mu_{ri})+\sum_{i=1}^{k_s}C_{si}(\mu_{si})+\\&+\chi\left[\frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)-T^*\right].\end{aligned} \tag{20}$$

The Lagrangian L contains $k_c + k_r + k_s + 1$ unknowns: χ; d_{ci}, $i=\overline{1,k_c}$; μ_{ri}, $i=\overline{1,k_r}$; μ_{si}, $i=\overline{1,k_s}$. The partial derivatives of L with respect to unknowns equalized to 0 are:

$$\begin{cases}\dfrac{\partial L}{\partial d_{ci}}=\dfrac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}+\dfrac{\chi}{\gamma}\cdot\dfrac{-\lambda_{ci}V_i}{\left(d_{ci}-\lambda_{ci}V_i\right)^2}=0,\ i=\overline{1,k_c}\\ \dfrac{\partial L}{\partial \mu_{ri}}=\dfrac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}+\dfrac{\chi}{\gamma}\cdot\dfrac{-\lambda_{ri}}{\left(\mu_{ri}-\lambda_{ri}\right)^2}=0,\ i=\overline{1,k_r}\\ \dfrac{\partial L}{\partial \mu_{si}}=\dfrac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}+\dfrac{g\chi}{\gamma}\cdot\dfrac{-\lambda_{si}}{\left(\mu_{si}-\lambda_{si}\right)^2}=0,\ i=\overline{1,k_s}\\ \dfrac{\partial L}{\partial \chi}=\dfrac{1}{\gamma}\left(\displaystyle\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)-T^*=0.\end{cases} \tag{21}$$

From the system of Eqs (21), provided that the restrictions of InEqs (1) are satisfied, it follows that the equalities must hold:

$$\left(d_{ci}-\lambda_{ci}V_i\right)^2\frac{\gamma}{\chi}\cdot\frac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}-\lambda_{ci}V_i=0,\ i=\overline{1,k_c}; \tag{22}$$

$$\left(\mu_{ri}-\lambda_{ri}\right)^2\frac{\gamma}{\chi}\cdot\frac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}-\lambda_{ri}=0,\ i=\overline{1,k_r}; \tag{23}$$

$$\left(\mu_{si}-\lambda_{si}\right)^2\frac{\gamma}{g\chi}\cdot\frac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}-\lambda_{si}=0,\ i=\overline{1,k_s}; \tag{24}$$

$$\frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}-\lambda_{si}}\right)-T^*=0. \tag{25}$$

Obtaining, based on Eqs. (22)-(25), the analytical expression for χ, and then for the expected capacities d_i ($i=\overline{1,k_c}$), μ_{ri} ($i=\overline{1,k_r}$), and μ_{si} ($i=\overline{1,k_s}$) does not succeed. But their values can be obtained, under certain conditions, using a numerical method for solving equations, for example the dichotomy method.

According to Eqs. (22)-(24) and taking into account the conditions of InEqs. (1) and the positive increasing character (by definition) of functions $C_{ci}(d_{ci})$ ($i = \overline{1,k_c}$), $C_{ri}(\mu_{ri})$ ($i = \overline{1,k_r}$), and $C_{si}(\mu_{si})$ ($i = \overline{1,k_s}$), the first factor of the sum of the left-hand side of Eqs. (22)-(24) at $\chi > 0$ is positive. So, the optimal solution is obtained at $\chi > 0$.

From Eqs. (22)-(24), as a result of some simple transformations, one obtains:

$$d_{ci} = \lambda_{ci}V_i + \sqrt{\frac{\chi\lambda_{ci}V_i}{\gamma\frac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}}},\ i=\overline{1,k_c}; \tag{26}$$

$$\mu_{ri} = \lambda_{ri} + \sqrt{\frac{\chi\lambda_{ri}}{\gamma\frac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}}},\ i=\overline{1,k_r}; \tag{27}$$

$$\mu_{si} = \lambda_{si} + \sqrt{\frac{\chi g\lambda_{si}}{\gamma\frac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}}},\ i=\overline{1,k_s}. \tag{28}$$

Similarly to Problem 1, the numerical solution of the system of Eqs. (25)-(28) is considerably simplified if all the ratios $\chi\Big/\frac{\partial C_{ci}(d_{ci})}{\partial d_{ci}}$ $(i=\overline{1,k_c})$, $\chi\Big/\frac{\partial C_{ri}(\mu_{ri})}{\partial \mu_{ri}}$ $(i=\overline{1,k_r})$, and $\chi\Big/\frac{\partial C_{si}(\mu_{si})}{\partial \mu_{si}}$ $(i=\overline{1,k_s})$, are increasing with respect to χ or all these ratios are decreasing with respect to χ. As in the case of Problem 1 (Scenario 3), we will examine only the situation in which each of the mentioned ratios is increasing or decreasing and, at the same time, the function

$$T(\chi)=\frac{1}{\gamma}\left(\sum_{i=1}^{k_c}\frac{\lambda_{ci}V_i}{d_{ci}(\chi)-\lambda_{ci}V_i}+\sum_{i=1}^{k_r}\frac{\lambda_{ri}}{\mu_{ri}(\chi)-\lambda_{ri}}+g\sum_{i=1}^{k_s}\frac{\lambda_{si}}{\mu_{si}(\chi)-\lambda_{si}}\right) \tag{29}$$

is increasing or decreasing with respect to χ.

Algorithm 4 for numerically solving the system of Eqs. (25)-(28) using the dichotomy method consists of the following.

1. Initial data: T^*; θ; γ; k_c; k_r; k_s; a; ε; ε_c; ε_r; ε_s; λ_{ci}, V_i, $C_{ci}(d_{ci})$, $i = \overline{1,k_c}$; λ_{ri}, $C_{ri}(\mu_{ri})$, $i = \overline{1,k_r}$; λ_{si}, $C_{si}(\mu_{si})$, $i = \overline{1,k_s}$.
2. Determine the character of the dependence $T(\chi)$. $\chi := 1$, $b := 1$.
   2.1. Determine the values of the capacities $d_{ci}(\chi)$, $i = \overline{1,k_c}$, $\mu_{ri}(\chi)$, $i = \overline{1,k_r}$, and $\mu_{si}(\chi)$, $i = \overline{1,k_s}$, based on the solution of Eqs. (26)-(28), taking into account the values of the quantities ε_c, ε_r, and ε_s.
   2.2. Determine $T(\chi)$ according to (29). $\Delta T := T(\chi) - T^*$. If $|\Delta T| < \varepsilon$, then go to Step 7.
   2.3. If b = 1, then $T_o(\chi) := T(\chi)$, $\chi := a\chi$ and go to Step 2.1.
   2.4. If $T(\chi) < T_o(\chi)$, then the function $T(\chi)$ is decreasing and go to Step 5.
3. Determine the interval $[\chi^-; \chi^+]$ at the increasing function $T(\chi)$.
   3.1. If $T_o(\chi) < T$, then $\chi^- := 1$ and go to Step 3.7.

3.2. $\chi^+ := 1$, $\chi := \chi/a$.

3.3. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on solving the Eqs. (26)-(28), taking into account the values of quantities ε_c, ε_r, and ε_s.

3.4. Determine $T(\chi)$ according to Eq. (29). $\Delta T := T(\chi) - T^*$. If $|\Delta T| < \varepsilon$, then go to Step 7.

3.5. If $\Delta T > 0$, then $\chi^+ := \chi$, $\chi := \chi/a$ and go to Step 3.3.

3.6. $\chi^- := \chi$, the interval $[\chi^-; \chi^+]$ is determined and go to Step 4.

3.7. If $T(\chi) > T^*$, then $\chi^+ := \chi$, the interval $[\chi^-; \chi^+]$ is determined and go to Step 4.

3.8. $\chi^- := \chi$, $\chi := a\chi$.

3.9. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on solving the Eqs. (26)-(28), taking into account the values of quantities ε_c, ε_r, and ε_s.

3.10. Determine $T(\chi)$ according to (29). $\Delta T := T(\chi) - T^*$. If $|\Delta T| < \varepsilon$, then go to Step 7.

3.11. If $\Delta T < 0$, then go to Step 3.8.

3.12. $\chi^+ := \chi$, the interval $[\chi^-; \chi^+]$ is determined.

4. Determine the solution, knowing the interval $[\chi^-; \chi^+]$ and taking into account that the function $T(\chi)$ is increasing.

4.1. $\chi := (\chi^- + \chi^+)/2$.

4.2. Determine the values of capacities $d_{ci}(\chi), i = \overline{1, k_c}$, $\mu_{ri}(\chi), i = \overline{1, k_r}$, and $\mu_{si}(\chi), i = \overline{1, k_s}$, based on solving the Eqs. (26)-(28), taking into account the values of the quantities ε_c, ε_r, and ε_s.

4.3. Determine the value of $T(\chi)$ according to Eq. (29). $\Delta T := T(\chi) - T^*$. If $|\Delta T| < \varepsilon$, then go to Step 7.

4.4. If $\Delta T > 0$, then $\chi^+ := \chi$, otherwise $\chi^- := \chi$ and go to Step 4.1.

5. Determine the interval $[\chi^-; \chi^+]$ similarly to Step 3, but taking into account that the function $T(\chi)$ is decreasing.

6. Determine the solution, knowing the interval $[\chi^-; \chi^+]$, similarly to Step 4, but taking into account that the function $T(\chi)$ is decreasing.

7. Determine C^* according to Eq. (18). The solution is obtained. ■

Some comments on Algorithm 4. Steps 3.7-3.12 determine χ^+, given that Step 3.1 determined the initial value of χ^-. Conversely, Steps 3.3-3.6 determine χ^-, given that Step 3.2 determined the initial value of χ^+. It should also be noted that the values of quantities ε_c, ε_r, and ε_s must be sufficiently small with respect to that of quantity ε, for Algorithm 4 to converge to the optimal solution.

Remark. Of course, the values of the quantities μ_{si} $(i = \overline{1, k_s})$, d_{ci} $(i = \overline{1, k_c})$, and μ_{ri} $(i = \overline{1, k_r})$, obtained according to Algorithm 4, are positive real numbers, and the capacities of the channels, routers and servers that could be used in the network belong to a finite set. Therefore, as in the case of Problem 1, an additional adaptation of the obtained solution is necessary.

The adjustment of the solution of Problem 3, so that the allowed capacities of the channels, routers and servers are used, is performed in a similar way to that described in Algorithms 2 and 3 for adjusting the solution of Problem 1 with the necessary adaptations, including the reversing the roles of quantities T and C.

4. Conclusions

Because of considerable expenses with the creation and development of computer networks, it is important their minimization. Moreover, sometimes it is useful the quick preliminary estimation of basic characteristics of the network. In this aim, taking into account the Jackson's partitioning theorem and considering the nonlinear dependence of the costs of channels, routers, and servers on their performance, a simplified analytical model of the backbone subnet and server set of wide area computer networks is defined. This model generalizes the similar model, but for linear dependence of the costs of channels, routers, and servers on their performance.

Based on this model, two optimization problems are formulated: minimizing the average response time to user requests of data processing and minimizing the summary cost of servers, channels, and routers of the computer network. For each of these two problems, by a simple algorithm, to obtain the solutions in continuous form regarding the necessary performances of channels, routers, and servers, is proposed.

Obtained in this way performances are positive real numbers. Usually, they do not coincide with the allowed performances of channels, routers, and servers, which take values from a range of discrete numbers. This is why, if necessary, for each of the two obtained solutions in continuous form, the problem of adjusting the obtained values of performances to the allowed discrete performances of the common-use network components, is formulated. To solve it, two algorithms are proposed: by reducing the initial problem to that of the backpack one and by using the resource concentration rule. The first of them, for the done solution in continuous form, is exact, but relatively complex, and the second one is simple, but not exact.

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