Journal of Engineering Science
Fascicle Electronics and Computer Science
Topic Electronics and Communication

Vol. XXXII, no. 2 (2025), pp. 26 - 34
ISSN 2587-3474
eISSN 2587-3482

https://doi.org/10.52326/jes.utm.2025.32(2).02
UDC 519.2:621.3.076.52

THE LIKELIHOOD FUNCTION BASED ON UNCENSORED/ CENSORED STATISTICAL DATA FOR MIN-PSD(MAX-PSD) AND MAX-PSD(MIN-PSD) AS LIFETIME DISTRIBUTIONS IN NETWORK RELIABILITY

Alexei Leahu *, ORCID: 0000-0002-1670-0111,
Veronica Andrievschi-Bagrin, ORCID: 0000-0001-8364-9873,
Maria Rotaru, ORCID: 0000-0002-3198-0297

Technical University of Moldova, 168 Stefan cel Mare Blvd., Chisinau, Republic of Moldova
* Corresponding author: Alexei Leahu, *alexei.leahu@ati.utm.md*

Received: 03. 11. 2025
Accepted: 04. 18. 2025

Abstract. In this paper general formulas for the likelihood function have been derived in the case when uncensored/censored statistical data refer to the lifetime of serial-parallel and parallel-serial type networks when the lifetimes of the system units are independent, identically distributed random variables, the number of subsystems and the number of units in each subsystem are random variables with power series type distribution. The formulas can be applied to obtain maximum likelihood estimators for the parameters of the lifetime distribution of the mentioned networks. The results are illustrated by examples of concrete probabilistic models.

Keywords: *lifetime distributions, Power Series Distribution, serial-parallel and parallel-serial networks, likelihood function, maximum likelihood estimator.*

Rezumat. În lucrare au fost deduce formule generale pentru funcția de verosimilitate în cazul în care datele statistice necenzurate/cenzurate se referă la durata de viață a rețelelor de tip serial-paralel și paralel-serial, când duratele de viață ale unităților sistemului sunt variabile aliatoare independente, distribuite identic, numărul de subsisteme și numărul de unități din fiecare subsistem sunt variabile aliatoare cu distribuție de tip serie de puteri. Formulele pot fi aplicate pentru a obține estimatori de verosimilitate maximă pentru parametrii distribuției duratelor de viață ale rețelelor menționate. Rezultatele sunt ilustrate prin exemple de modele probabilistice concrete.

Cuvinte cheie: *distribuția duratei de viață, Distribuție de tip Serie de Puteri, rețele serial-paralelele și paralel-seriale, funcția de verosimilitate, estimator de verosimilitate maximă.*

1. Introduction

The problem of obtaining maximum likelihood estimators for the lifetime distribution parameters of serial-parallel and parallel-serial networks first requires knowledge of the likelihood function based on both uncensored and censored statistical data. Since dynamic probabilistic models have already been launched and researched for the mentioned networks, following which the most general analytical formulas were obtained [1], it is natural that they have a similar continuity in the case of the likelihood function.

2. Auxiliary notions and results

Here are the results from [1] that we will continue to rely on. It is about the two networks type *A*, serial-parallel and type *B*, parallel-serial [1], [2], according to Figure 1.

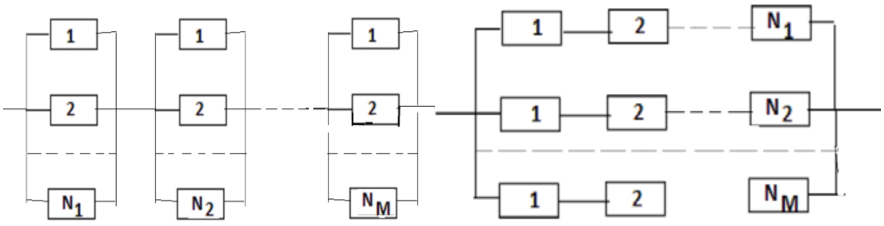
The general probabilistic model, in the case of both networks, assumes the following:

-the lifetimes of the network units are non-negative, independent, identically distributed random variables (i.i.d.r.v.) with the cumulative distribution function (c.d.f.) $F(x) = F_X(x, \lambda)$, where parameter λ ∈⁄ ⊆ $\mathbf{R}^{\mathrm{k}}$;

-the number *M* of subnets is a r.v. with possible values from the set of natural numbers, of the Power Series Distributions class *(*PSD) [3], [4} with the power series function $B(\omega) = \sum_{m\geq 1} b_m \omega^m$, with the radius of convergence, τ > 0, i.e., $P(M = m) = \frac{b_m \omega^m}{B(\omega)}, b_m \geq 0, m = 1,2,\dots,\omega \in (0,\tau)$;

- the numbers N_k of units in the subnets k = 1, 2, . . . , M are *0*-truncated , i.i.d.r.v., of class *PSD*, with power series function, $A(\theta) = \sum_{n\geq 1} a_n \theta^n$, with the radius of convergence, τ > 0, i.e., $P(N_k = n) = \frac{a_n \theta^n}{A(\theta)}, a_n \geq 0, n = 1,2,\dots,\theta \in (0,\tau)$;

-the lifetimes of the network units and the numbers *M, N_k, k =1, 2, . . . , M* are completely independent r.v.



a) b)

Figure 1. Schematic representation of serial- parallel and parallel-serial networks: a) Serial-Parallel Network scheme; b) Parallel-Serial Network scheme [1].

From [1] we will call on the following:

Propositon. *The cumulative distribution functions* $U(x;\ \lambda,\propto,\omega))$ *and* $V(x;\ \lambda,\propto,\omega))$ *of the lifetimes of the networks, respectively, of serial-parallel type and parallel-serial type, can be calculated according to the formulas:*

$$\boldsymbol{U(x;\ \lambda,\propto,\omega) = 1 - \frac{B(\omega(1-A(\theta F(x;\lambda))\ /\ A(\theta)\,)}{B(\omega)}} \tag{1}$$

$$\boldsymbol{V(x;\ \lambda,\propto,\omega) = \frac{B(\omega(1-A(\theta(1-F(x;\lambda)))\ /\ A(\theta)\,)}{B(\omega)}} \tag{2}$$

Remark. Because c.d.f. $U(x;\ \lambda,\propto,\omega))$, as the lifetime distribution of serial-parallel network,

coincides with c.d.f. of the r.v. $min(\max_{1\leq i\leq N_1} X_{i1}, \max_{1\leq i\leq N_2} X_{i2},\dots, \max_{1\leq i\leq N_M} X_{iM})$ and $V(x;\ \lambda,\propto,\omega))$, as the lifetime distribution of the parallel-serial network, coincides with c.d.f. of the r.v. $max(\min_{1\leq i\leq N_1} X_{i1}, \min_{1\leq i\leq N_2} X_{i2},\dots, \min_{1\leq i\leq N_M} X_{iM})$, where X_{ij} , *i=1, ..., Nj* , *j=1, ..., M* are lifetimes of all units, r.v. *M ∈ PSD* with power series function *B(ω)* and $N_1, N_2, \dots, N_M$ are i.i.d.r.v. as r.v. *N ∈ PSD* with power series function $A(\vartheta)$, the distributions $U(x;\ \lambda,\propto,\omega))$ and $V(x;\ \lambda,\propto,\omega))$ will be called, respectively, *Min-PSD(Max-PSD)* and *Max-PSD(Min-PSD)* lifetime distributions in network reliability [1].

These general formulas allow us to determine, in practice, the likelihood function for any concrete serial-parallel or parallel-serial type network model, considered as a particular case of the models described above. We note, that from the Ec. (1)-(2) we have the following result [1].

Consequence. *If the lifetime of each unit is a r.v. of (absolutely) continuous type, then the lifetimes of the respective networks will be r.v. of (absolute) continuous type too. At the same time, if the lifetime of each unit is a r.v. of discrete type, then the lifetimes of the respective networks will be r.v. of discrete type too.*

3. Likelihood function based on uncensored data

Let us consider the sample of size *n* of uncensored data $(x_1, x_2, \ldots, x_n)$ of the lifetimes of a network of type *A* or *B*. The fact that the data are uncensored means that the values $x_1, x_2, \ldots, x_n$ represent the results of *n* observations made on the lifetime of the network from the start of its operation until its failure. The likelihood function is defined based on the Maximum Likelihood Principle, according to which: if a random event has occurred, then it means that this is the event with the highest probability to occur.

In the assumption that c.d.f. of the lifetime of each unit of the network depends on a parameter, let's say, λ, i.e., $F(x) = F(x;\lambda)$. Then Ec. (1) − (2) show that the distributions of network lifetimes, generally, depend on 3 parameters, λ, θ, ω.

Case1. *The lifetime of each units is a r.v. of (absolute) continuous type*

Most of the probabilistic models that aim at the lifetime of the networks we approach start from the assumption that the lifetime of each unit is a r.v. of absolute continuous type. So, according to our Consequence, the lifetime of the network will also be a r.v. of absolute continuous type. This means that, having the Ec. (1) − (2) at our disposal, we can determine the probability density function (p.d.f.) of the lifetimes for serial-parallel networks and serial-parallel networks, respectively, according to the formulas:

$$u(x;\ \lambda,\propto,\omega\,) = \frac{dU(x;\lambda,\propto,\omega\,)}{dx},\quad v(x;\ \lambda,\propto,\omega\,) = \frac{dV(x;\lambda,\propto,\omega\,)}{dx}.$$

This means that the respective likelihood functions will be written as follows:

$$\boldsymbol{L_U(x_1, x_2\,, \ldots, x_n;\ \lambda,\propto,\omega\,) = \prod_{k=1}^{n} u(x_k;\lambda,\propto,\omega\,)} \tag{3}$$

$$\boldsymbol{L_V(x_1, x_2\,, \ldots, x_n;\ \lambda,\propto,\omega\,) = \prod_{k=1}^{n} v(x_k;\lambda,\propto,\omega\,)} \tag{4}$$

Example 1. We will take as a special case of the models described by us, the case when the number of subnetworks *M* is not random, but is constant, being equal with natural number *M* , and the number of units N_i in the subnetwork number $i = 1, \ldots, M$ is also constant , being known and equal to a natural number *N*. Therefore, formally, we can consider that *M* is of the *PSD* class with the power series function $B(\omega) = \omega^M$, but also that *N* is of the *PSD* class with the power series function $A(\vartheta) = \vartheta^N$. We assume, for example, that the lifetime of each unit is exponentially distributed r.v. with parameter λ > 0, i.e., with c.d.f. $F(x)=(1-e^{-\lambda x})I_{[0,+\infty]}(x)$.

The problem arises of constructing a maximum likelihood estimator (m.l.e.) for this parameter, having available the experimental data of the lifetimes *(x_1, x_2, ..., x_n)* aimed at the results of the operation of *n* identical serial -parallel or parallel-serial networks.

Solution. Substituting in the Ec. (1)-(2) the concrete expressions of the functions *F(x), A(ϑ)* and *B(ϑ)*, we deduce that in this case the lifetime c.d.f. of the serial-parallel and parallel-serial network are, respectively,

$$\boldsymbol{U}(\boldsymbol{x};\boldsymbol{\lambda},\mathbf{N},\mathbf{M}) = \mathbf{1} - \left(\mathbf{1} - \left(\boldsymbol{F}(\boldsymbol{x})\right)^{\boldsymbol{N}}\right)^{\boldsymbol{M}} = \mathbf{1} - \left(\mathbf{1} - \left(\mathbf{1}-\boldsymbol{e}^{-\boldsymbol{\lambda} \boldsymbol{x}}\right)^{\boldsymbol{N}}\right)^{\boldsymbol{M}} \boldsymbol{I}_{[\mathbf{0},+\infty)}(\boldsymbol{x}) \tag{5}$$

$$V\,(x;\,\lambda, N\,, M) \;=\; \left[1 \;-\; \left(1 \;-\; F\,(x)\right)^{N}\right]^{M} \;=\; (1 - e^{-\lambda N x})^{M} I_{[0,+\infty)}(x) \tag{6}$$

According to the above Consequence, because the lifetime of each unit is (absolutely) continuous r.v., we deduce that the lifetimes of our networks are (absolutely) continuous r.v. with, respectively, probability density functions

$$u(x;\lambda, N\,, M) = \frac{dU}{dx} = (\mathrm{MN}\lambda)e^{-\lambda x}(1-e^{-\lambda\, x})^{N-1}\,(1 - (1-e^{-\lambda\, x})^{N})^{M-1} I_{[0,+\infty]}(x)$$

$$v(x;\lambda, N\,, M) = \frac{dV}{dx} == \mathrm{MN}\lambda e^{-\lambda N x}\left(1\, - e^{-\lambda N x}\,\right)^{\mathrm{M}\,-1} I_{[0,+\infty]}(x)$$

So, Likelihood function, corresponding to the continuous data, for serial-parallel network is

$$L_U(x_1, x_2\,, \dots, x_n;\; \lambda, N\,, M) = \prod_{k=1}^{n} u(x_k;\lambda, N\,, M) =$$

$$(MN\lambda)^n e^{-\lambda N \sum_{k=1}^{n} x_k} * \prod_{k=1}^{n}\left(1-e^{-\lambda\, x_k}\right)^{N-1}\,((1 - (1-e^{-\lambda\, x_k})^{N})^{M-1}$$

and for parallel-serial network is

$$L_V(x_1, x_2\,, \dots, x_n;\; \lambda, N\,, M) = \prod_{k=1}^{n} v(x_k;\lambda, N\,, M) =$$

$$(MN\lambda)^n e^{-\lambda N \sum_{k=1}^{n} x_k} \prod_{k=1}^{n}\left(1-e^{-\lambda\, N x_k}\right)^{M-1}$$

By the definition, the maximum likelihood estimator (m.l.e.) for the parameter λ, parameters *M, N* being known, represents that value $\hat{\lambda}$ for which the likelihood function takes its maximum value (see Maximum Likelihood Principle) [5]. For *M > 1, N>1* , our likelihood functions can be maximized using numerical methods only, but for *M = 1,* otherwise, when the parallel-serial network is always more reliable than the serial-parallel network, this problem can be explicitly solved for the parallel-serial network. Indeed, in this case, the maximum likelihood estimator (m.l.e.) $\hat{\lambda}$ for the parameter *λ*, our likelihood function can be maximized, solving likelihood equation:

$$\frac{d}{d\lambda} lnL_V(x_1, x_2\,, \dots, x_n;\; N, \lambda) = 0$$

i.e., equation

$$\frac{d}{d\lambda}\left[n(lnN + ln\lambda) - \sum_{k=1}^{n} \lambda N x_k\right] = 0 \quad \text{i.e.,}$$

$$\frac{n}{\lambda} - N\sum_{k=1}^{n} x_k = 0$$

In this way, for paralel-serial network, we find that

$$m.l.e.\hat{\lambda} = \frac{n}{N\sum_{k=1}^{n} x_k}$$

Case 2. ***The lifetime of each units is a r.v. of discrete type***

The Ec. (1) - (2) being valid also when the lifetime of the network units is r.v. of discrete type, then, according to our Consequence, the lifetimes of the networks will of discrete type too. More precisely, if the lifetime *X* of each unit is this, for example, a r.v. with values from the set {0,1,2, ..., k, ...} given by the parametric probabilistic distribution $P_\lambda\,(X = k)$, where $P_\lambda\,(X = k) \geq 0,\ k = 0,1,2,\ldots,\ \sum_{k\geq o} P_\lambda\,(X = k) = 1$, then the lifetimes *U* and *V* of our networks will also be r.v. of discrete type too, with the possible values from the same set. Because for any integer value *x* from the set of possible values, for example, for r.v. *U*, we have that c.d.f. is equal with $U(x;\ \lambda, \propto, \omega) = \sum_{k:k\leq x} P_{\lambda,\propto,\omega}\,(U = k)$, it turns out that for the lifetime *U* of the serial-parallel network

$$P_{\lambda,\propto,\omega}\,(U = 0) = \mathrm{U}(0;\ \lambda, \propto, \omega), \quad \textit{and}$$
$$P_{\lambda,\propto,\omega}\,(U = k) = U(k;\ \lambda, \propto, \omega) - U(k-1;\ \lambda, \propto, \omega), for\ k \geq 1 \tag{7}$$

Analogously, for the lifetime *V* of the parallel-serial type network

$$P_{\lambda,\propto,\omega}\,(V = 0) = V(0;\ \lambda, \propto, \omega), \quad \textit{and}$$
$$P_{\lambda,\propto,\omega}\,(V = k) = V(k;\ \lambda, \propto, \omega) - V(k-1;\ \lambda, \propto, \omega), for\ k \geq 1 \tag{8}$$

According to the Maximum Likelihood Principle in Case 2, using Ec. (7) - (8), the respective functions for serial-parallel and parallel-serial networks will be written as follows:

$$L_U(x_1, x_2, \ldots, x_n;\ \lambda, \propto, \omega) = \prod_{k=1}^{n} P_{\lambda,\propto,\omega}\,(U = x_k). \tag{9}$$

$$L_V(x_1, x_2, \ldots, x_n;\ \lambda, \propto, \omega) = \prod_{k=1}^{n} P_{\lambda,\propto,\omega}\,(V = x_k) \tag{10}$$

where $P_{\lambda,\propto,\omega}\,(U = x_k)\ and\ P_{\lambda,\propto,\omega}\,(V = x_k)$ are calculated according to the Ec. (7) – (8).

Example 2. We will consider the same model as in Example 1, with the difference that the lifetimes are random variables, independent, identically distributed geometrically with the parameter λ, 0<λ<1, i.e., lifetime is a r.v. given by distribution

$$P_\lambda\,(X = k) = \lambda(1-\lambda)^k, k = 0,1,2, \ldots$$

So, for each *x, x=0,1,2, ...,* c.d.f.

$$F(x;\ \lambda) = \sum_{k=0}^{x} \lambda(1-\lambda)^k = 1 - (1-\lambda)^{x+1}$$

Using Ec. (5) – (6) we find that

$$P_{\lambda,,N,M}\,(U = k) = (1-(1-(1-\lambda)^k)^N)^M - (1-(1-(1-\lambda)^{k+1})^N)^M, for\ k \geq 0$$

Replacing these probabilities, respectively, in the Ec. (9) – (10), we obtain the corresponding Likelihood Functions for serial-parallel and parallel-serial networks. As they show, $m.l.e.\hat{\lambda}$ can only be found by numerical methods.

4. Likelihood function based on censored data

In Statistics data are called censored when the value of an observation is partially known. This is usually the case in survival/reliability analysis, where the time to a certain event is of interest, but for some studies, the event has not yet occurred at the time of analysis. For example, if we are studying the lifetime of a product, the censored data would be cases where the product is still working at the end of the study period, so we do not have an exact value for lifetime.

There are several types of censorship [6,7]:

- ✓ **Right-censoring.** We don't know what happens after a certain point.

- ✓ **Left censoring.** Left-censored observations occur in life test applications when a system has failed at the time of its first inspection; all that is known is that the unit failed before the inspection time. We have no information about what happened before a certain point.
- ✓ **Interval censoring.** We know that the event occurred within a certain interval, but we do not know the exact time.

Censoring is important because it affects how we analyze and interpret data. Statistical methods must be adapted to account for the incomplete information provided by censored data. It is important for us to know that in the case of censored data, the Likelihood Function is written more simply, because regardless of the type of data (discrete or (absolutely) continuous), we only need the c.d.f. of the observed variable.

Regarding our case, when the lifetime *Y*, given by c.d.f. $F_Y(x)$, targets serial-parallel or parallel-parallel networks, the censored data is represented by random events of the form: $\{Y \leq a\}$, i.e., *the data is left censored*; $\{a < Y \leq b\}$, i.e., *the data is interval censored*, $\{Y > b\}$, i.e., *the data is right censored*, where $0<a<b<+\infty$. Otherwise, since the lifetime is a non-negative r.v., we note that both left censoring and right censoring can be considered special cases of interval censoring. Indeed: probabilities of these events are equals to

$$P\{Y \leq a\} = F_Y(a),\ P\{a < Y \leq b\} = F_Y(b) - F_Y(a), P\{Y > b\} = 1 - F_Y(b).$$

So, we may solve the problem of writing the Likelihood Function. For this, it is sufficient to know the following information:

(1) the *a* and *b*, as the values that determine the 3 types of data censoring;

(2) the c.d.f. $F_Y(x)$ of the lifetime *Y*, also known as a function, that depends on 3 parameters, more exactly, $F_Y(x) = F_Y(x;\ \lambda, \propto, \omega)$;

(3) the probabilities $P\{Y \leq a\}, P\{Y > b\}, P\{a < Y \leq b\}$;

(4) the sample size *n* of data and the numbers n_1, n_2 and n_3, respectively, of the left, interval and right censored data, where $n_1+n_2+n_3=n$.

Then, according to the Maximum Likelihood Principle, Likelihood Function corresponding to the censored data of lifetime *Y* is done by the formula:

$$L_Y(n_1, n_2, n_3\,; \lambda, \propto, \omega) =$$
$$[F_Y(a;\ \lambda, \propto, \omega)]^{n_1}] * [1 - F_Y(a;\ \lambda, \propto, \omega)]^{n_2} * [F_Y(\mathrm{b};\ \lambda, \propto, \omega) - F_Y(\mathrm{a};\ \lambda, \propto, \omega)]^{n_3}.$$

Replacing in this formula c.d.f. $F_Y(x;\ \lambda, \propto, \omega)$ with c.d.f. $U(x;\ \lambda, \propto, \omega)$ or c.d.f. $V(x;\ \lambda, \propto, \omega)$, given by the Ec. (1)-(2), we obtain Likelihood Functions for serial-parallel and parallel-serial networks.

Remark. The above likelihood function represents the case when the data are censored on 3 intervals, but it can be extended, similarly, to the case when the number of censoring intervals is greater than 3. This will be illustrated in Exemple 3.

Example 3. We will consider the same model as in Example 1, with the difference that we have *n* cenzored data, where number of left cenzored data with the threshold *a* is equal with n_1, the number of censored data by interval *(a,b]* is equal with n_3 and number of right cenzored data with the threshold *b* is equal with n_3, $a>0$, $a<b<+\infty$, $n_1+n_2+n_3=n$. Then, on the base of Ec. (11), using, respectively Ec. (5)-(6), the Likelihood Functions for serial-parallel and parallel networks may by write, respectively, as

$$L_U(n_1, n_2, n_3\,;\ \lambda, N, M =$$

$$\left(\frac{\boldsymbol{n}!}{\boldsymbol{n_1}!\,\boldsymbol{n_2}!\,\boldsymbol{n_3}!}\right) * \left\{\mathbf{1} - (\mathbf{1} - \left(\mathbf{1} - \boldsymbol{e}^{-\lambda a})^{\boldsymbol{N}}\right)^{\boldsymbol{M}}\right\}^{\boldsymbol{n_1}} \left\{[(\mathbf{1} - \left(\mathbf{1} - \boldsymbol{e}^{-\lambda a})^{\boldsymbol{N}}\right)]^{\boldsymbol{M}} - [(\mathbf{1} - \left(\mathbf{1} - \boldsymbol{e}^{-\lambda b})^{\boldsymbol{N}}\right)]^{\boldsymbol{M}}\right\}^{\boldsymbol{n_2}} * \{[(\mathbf{1} - \left(\mathbf{1} - \boldsymbol{e}^{-\lambda b})^{\boldsymbol{N}}\right)]^{\boldsymbol{M}}\}^{\boldsymbol{n_3}} \tag{11}$$

$$\boldsymbol{L_V}(\boldsymbol{n_1}, \boldsymbol{n_2}, \boldsymbol{n_3};\ \boldsymbol{\lambda}, \boldsymbol{N}, \boldsymbol{M}) = \left(\frac{\boldsymbol{n}!}{\boldsymbol{n_1}!\,\boldsymbol{n_2}!\,\boldsymbol{n_3}!}\right)\left[\left(\mathbf{1} - \boldsymbol{e}^{-\lambda N a}\right)^{\boldsymbol{M}}\right]^{\boldsymbol{n_1}} * [\left(\mathbf{1} - \boldsymbol{e}^{-\lambda N b}\right)^{\boldsymbol{M}} - \left(\mathbf{1} - \boldsymbol{e}^{-\lambda N a}\right)^{\boldsymbol{M}}]^{\boldsymbol{n_2}} * \left[\mathbf{1} - \left(\mathbf{1} - \boldsymbol{e}^{-\lambda N b}\right)^{\boldsymbol{M}}\right]^{\boldsymbol{n_3}} \tag{12}$$

To find out the m.l.e. $\hat{\lambda}$ for the parameter λ we will take the case when the parallel-serial networkis more reliable than serial parallel, i.e., the case when, according to the work [1], *N<M*. For example: *N=2, M=3*, and as the unit of lifetime we will take *1* year. To test the algorithm for obtaining a maximum likelihood estimate for the parameter λ, we simulate Monte Carlo [8]-[15], using Chat GPT 4, values of the lifetime of our network in the case, for example, when λ =0.2, i.e., when the lifetime . of each unit in the network has an average value equal to 1/ λ =5 years. Here are the simulated values written as a variational string in ascending order:

$$(1.23,1.67,1.89, 2.34,2.56,2.78,2.89,3.12,3.45,3.78,4.01,4.23,4.56,5.01,5.34,5.67,5.89, 6.12,6.45,6.78).$$

We assume that these data are censored according to the values *a=2 years* and *b=4 years*. So, we assume that we have at our disposal, from a total number of *n = 20* data, $n_1=3$ data censored on the left, $n_2=7$ data censored on the interval (2,4] and $n_3=10$ data censored on the right. Then the Likelihood Function is

$$L_V(3, 7{,}10;\ \lambda, 2, 3) = \frac{20!}{3!\,7!\,10!} * ((1 - e^{-4\lambda})^3)^3 * [\left(1 - e^{-8\lambda}\right)^3 - \left(1 - e^{-4\lambda}\right)^3]^7 * [1 - \left(1 - e^{-8\lambda}\right)^3]^{10}$$

By means of the Mathematica 14.0 we find that value of λ for which the Likelihood Function takes its global maximum, that is, we find that m.l.e. $\hat{\lambda}$=0.0882487. Comparing it with the true value of λ=0.2, we find that the approximation is not so good. So it's a problem related to the nature of censorship.

Now, on the base of the above simulated data, we assume that these data are censored according to the values *a=2 years*, *b=4 years* and *c=6 years*. That means we assume that we have at our disposal, from a total number of *n = 20* data, $n_1=3$ data censored on the left, $n_2=7$ data censored on the interval (2,4], $n_3=7$ data censored on the interval (4,6], and $n_4=3$ data censored on the right. If in the previous case we had data censored on 3 intervals, now we will have to deal with data censored on 4 intervals. The method of calculating the Likelihood Function being similar, we find that

$$L_V(3, 7{,}7{,}3;\ \lambda, 2, 3) = \frac{20!}{3!7!7!3!} * ((1 - e^{-4\lambda})^3)^3 * ((1 - e^{-8\lambda})^3 - \left(1 - e^{-4\lambda}\right)^3)^7 * * (1 - \left(1 - e^{-12\lambda}\right)^3)^3$$

Now, also by means of Mathematica 14.0, we find that value of λ for which the Likelihood Function takes its global maximum, i.e., we find that m.l.e. $\hat{\lambda}$ =0.214796 satisfactorily approximates the true value of the parameter λ=0.2.

5. Conclusions

General formulas in Eq. (1)-(2) for determining c.d.f. of lifetimes is a large source of dynamic probabilistic models for serial-parallel or parallel-serial networks, but also a basis for writing the Likelihood Function, when the data are uncensored or censored. Writing the likelihood function for censored data becomes simpler, because it does not depend on the type of lifetime as r.v. (discrete or continuous), using only c.d.f. But the problem of finding an m.l.e. which approximates as well as possible the true value of the unknown parameter is complicated, because it is difficult to match the censoring intervals. The fact that matching the censoring intervals is difficult, even when we rely on simulation data, shows us that in the case of real problems the use of maximum likelihood estimators must be done with great caution if the choice of censoring intervals does not have a mathematical reasoning. However, this is a problem that deserves to be researched more deeply. The examples given show that finding the maximum likelihood estimators becomes a maximization problem that can be solved, as a rule, by numerical methods.

These results were presented at the International Conference on Electronics, Communications and Computing, ECCO 2024, 17-18 October, Chisinau, Republic of Moldova.

Acknowledgments: The results were obtained within the Institutional Research Project 020404 concluded with the Ministry of Education and Research of the Republic of Moldova.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Leahu, A.; Andrievschi-Bagrin, V.; Ciorbă, D.; Fiodorov, I. On dynamic probabilistic models in network reliability. In: *Changes and Innovations in Social Systems,* Hoscova-Mayerova, S.; Flaut, C.; Flaut, D.; Rascova, P. (Eds); Springher Nature, Cham, Switzerland, 2025, 657 p.
2. Kapur, K.C.; Lamberson, L.R. *Reliability in engineering design.* Wiley India Pvt. Limited, New Delhi, India, 2009, 608 p.
3. Noack, A. A class of random variables with discrete distributions. *Annals of Mathematical Statistics* 1950, 21, pp. 127–132.
4. Leahu, A.; Andrievschi-Bagrin, V.; Ciorbă, D., Fiodorov, I. Once again about the reliability of serial-parallel and parallel-serial networks. In: *International Conference on Electronics, Communications and Computing*, 21-22 October, 2021, Chisinau, Republic of Moldova, pp. 170-173.
5. Barlow, R.; Proshan, F. *Statistical Theory of Reliability and Life testing: Probability Models*. Holt, Rinehart & Winston Inc., NY, USA, 1974, 290 p.
6. Meeker, W. Q.; Escobar, L.A. *Statistical Methods for Reliability Data.* John Wiley & Sons, Inc., NY, SUA, 1989, 680 p.
7. Lee, E. T.; Wang, J. W. Statistical Methods for Survival Data Analysis, 3rd Edition. John Wiley&Sons, New Jersey, USA 2003, 513 p.
8. Gertsbakh, I.; Spungin Y. *Models of Network Reliability: Analysis, Combinatorics and Monte Carlo*, CRC Press, Boca Raton, USA, 2012, 217 p.
9. Gertsbakh, I. *Reliability Theory with Applications to Preventive Mentenance*. Springer-Verlag Berlin Heidelberg, Germany, 2005, 215 p.
10. Kroese, D.; Taimre, T.; Botev, Z. I. *Handbook of Monte Carlo Methods*. John Wiley & Sons, Inc., New Jersey, USA, 2011, 727 p.
11. Ross, S. M. *Introduction to Probability Models*. 10 th Edition. Elsevier, Amsterdam, Netherlands, 2010, 784 p.
12. Faulin, J.; Juan, A.A.; Martorell, S.; Ramires-Marquez, J.-E. *Simulation Methods for Reliability and Availability of Complex Systems*. Springer-Verlag London Limited, London, UK, 2010, 315 p.
13. Hoang Pham (Editor). *Handbook of Reliability Engineering*. Springer-Verlag London, London, UK, 2003, 663 p.
14. O'Connor, P.T.D.; Kleyner, A. *Practical Reliability Engineering*, 5th Editions. John Wiley&Sons, New Delhi, India, 2012, 484 p.
15. Barbu, A.; Song-Chun Zhu. *Monte Carlo Methods*. Springer Nature, Singapore Pte Ltd, Singapore, 2020, 422 p.

Citation: Leahu, A.; Andrievschi-Bagrin, V.; Rotaru, M. The likelihood function based on uncensored/ censored statistical data for Min-PSD(Max-PSD) and Max-PSD(Min-PSD) as lifetime distributions in network reliability. *Journal of Engineering Science*. 2025, XXXII (2), pp. 26-34. https://doi.org/10.52326/jes.utm.2025.32(2).02.

Submission of manuscripts: jes@meridian.utm.md