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TUNING THE CONTROLLER FOR OBJECT MODELS WITH ONE TO FOUR POLES USING THE POLINOMIAL METHOD

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Abstract. The paper describes the procedure for tuning the controller for control object models with one to four poles using the polynomial method, by imposing the damping ratio and the settling time of the synthesized system. The numerator and denominator polynomials of the object's transfer function are decomposed into components containing zeros in the left and right halves of the complex plane. Based on the order of the object model and the conditions for solving the system of algebraic equations, the physical feasibility of the control algorithm, and the robustness of the system, the desired polynomial of the synthesized system is constructed. This polynomial consists of two sub-polynomials with unknown coefficients, and the degrees and unknown polynomials as well as the degree of the desired polynomial are calculated. From the damping ratio and settling time, the dominant poles of the synthesized system are determined. Based on these poles, the characteristic polynomial of the closed-loop system is constructed, and, if necessary, additional real poles are added as far away as possible from the dominant poles to meet the system's performance requirements. From the resulting equality, by equating the coefficients of the same powers of the variable s on the left and right sides of the equality, a system of algebraic equations is obtained, from which the unknown coefficients and polynomials are determined. Using the stable components of the object model and the unknown polynomials, the transfer function of the control algorithm is constructed. Examples of controller tuning for first-fourth order object models using the polynomial method have been analyzed. The synthesized systems have high performance and good robustness.

Keywords: *models of systems with first- to fourth-order inertia, transfer function, controller, tuning parameters, control system, tuning methods, controller tuning, system response, system performance, robustness.*

Abstract: În lucrare se prezintă procedura de acordare a regulatorului la modele de obiecte de conducere cu inerție de ordinul unu-patru după metoda polinomială cu impunerea gradului de amortizare și timpului de reglare ale sistemului sintetizat. Polinoamele numărătorului și numitorului funcției de transfer a obiectului se descompun în componente care conțin zerourile în partea stângă și partea dreaptă a planului complex. În baza ordinului modelului obiectului și din condițiile de soluționare a sistemului de ecuații algebrice, realizabilitatea fizică a algoritmului de conducere și robustețea sistemului, se construiește polinomul dorit al sistemului sintetizat, care conține două polinoame cu coeficienții necunoscuți și se calculează gradele și polinoamele necunoscute și gradul polinomului dorit. După gradul de amortizare și timpul de reglare se determină poliile dominante ai sistemului sintetizat și în baza lor se construiește polinomul caracteristic al sistemului închis și, după necesitate, se adaugă poli reali suplimentari alocați cât mai departe de poliile dominante pentru a satisface performanțele sistemului. Din egalitatea obținută, prin egalarea coeficienților de pe lângă aceleași puteri ale variabilei s din partea stângă și partea dreaptă a egalității, se obține un sistem de ecuații algebrice, din care se determină coeficienții și polinoamele necunoscute. În baza componentelor stabile ale modelului obiectului și polinoamelor necunoscute, se construiește funcția de transfer a algoritmului de conducere. S-au analizat exemple de acordare a regulatorului pentru modele de obiecte de gradul unu-patru prin metoda polinomială. Sistemele sintetizate au performanțe ridicate și robustețe bună.

Cuvinte cheie: modele de obiecte cu inerție de ordinul unu-patru, funcție de transfer, regulator, parametrii de acord, sistem automat, metode de acordare, acordarea regulatorului, răspunsul sistemului, performanțele sistemului, robustețe.

1. Introduction

In the automation of various technical objects, industrial and technological processes that require automatic control [1-6], whose evolution is approximated by mathematical models described through transfer functions with constant parameters. Depending on the properties of the processes, the mathematical models associated with these processes will have the respective structure and corresponding order.

There is a wide variety of technical objects (e.g., automobiles, spacecraft, telescopes, plotters, lasers, elevators, nuclear reactor electrodes, linear drives, etc.) [1-6] and technological processes [1,3,4,6] described by mathematical models with inertia of degree $n = 1, 2, 3, 4$ in the form of transfer functions:

$$H_1(s) = \frac{b_0}{a_0s + a_1} = \frac{B(s)}{A(s)}, \quad (1)$$

$$H_2(s) = \frac{b_0}{a_0s^2 + a_1s + a_2} = \frac{B(s)}{A(s)}, \quad (2)$$

$$H_3(s) = \frac{b_0}{a_0s^3 + a_1s^2 + a_2s + a_3} = \frac{B(s)}{A(s)} \quad (3)$$

$$H_4(s) = \frac{b_0}{a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4} = \frac{B(s)}{A(s)}, \quad (4)$$

where $b_0 = k$ is the transfer coefficient, a_0, a_1, a_2, a_3, a_4 – time constants.

For models (1)-(4), there are methods for tuning the controller, such as the pole-zero method, frequency methods, integral criteria methods, etc. However, the use of these methods is accompanied by complex calculations to achieve the desired performance [8-11].

The paper describes the synthesis of the control algorithm for models (1)-(4) using the polynomial method, ensuring that the synthesized system achieves high performance and good robustness [12-17].

2. The Controller Tuning Algorithms

In the study, the structural diagram of the system is used (Figure 1), composed of the controller described by the transfer function $H_R(s)$ and the control object model with the transfer function $H(s)$, where $r(t)$ is the reference signal, $\varepsilon(t)$ – the system error, $u(t)$ – the control signal, $y(t)$ – the system output.

The automatic system with the damping factor $\xi = \frac{\sqrt{2}}{2} = 0.707$ has the highest performance [1-4]. For the synthesis of the control algorithm using the polynomial method, the damping ratio is imposed $\xi = 0.707$ and $\xi = 0.8$, and the settling time $t_s = 1$ s.

The natural frequency and the dominant poles of the synthesized system are determined [3,4]:

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.707 t_s}, \quad (5)$$

$$p_{1,2} = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2} = -\alpha \pm j \omega, \quad (6)$$

with $\alpha = \xi \omega_n$ - the real part and $\omega = \omega_n \sqrt{1 - \xi^2}$ - the imaginary part of the root - the frequency.

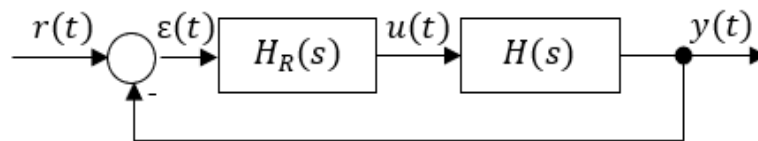


Figure 1. Structural diagram of the automatic system.

According to the polynomial method, the transfer function of the object is described in factorized form [13,14]:

$$H(s) = \frac{B(s)}{A(s)} = \frac{B^-(s)B^+(s)}{A^-(s)A^+(s)}, \quad (7)$$

where: $B^-(s)$, $A^-(s)$ are polynomials with zeros allocated in the left half of the plane, and $B^+(s)$, $A^+(s)$ are polynomials with zeros allocated in the right half of the plane and neutral zeros allocated at the origin of the complex plane. If the polynomials $B(s)$ and $A(s)$ do not contain left-hand zeros, then the polynomials $B^-(s)$, $A^-(s)$ are equal to constants, and if the polynomials $B(s)$ and $A(s)$ do not contain straight zeros, then the polynomials $B^+(s)$, $A^+(s)$ are equal to one. The degrees of the respective polynomials are denoted: n_{B^-} , n_{B^+} , n_{A^-} , n_{A^+} , $n_A = n$.

The desired characteristic polynomial of the synthesized system is constructed in the form:

$$B^+(s)M(s) + A^+(s)N(s)s^r = G(s), \quad (8)$$

where: $M(s)$ and $N(s)$ are unknown polynomials that will be determined, and the component s^r introduces a degree of astatism of r in the system structure to achieve zero steady-state error $\varepsilon = 0$ and the degrees of the polynomials are denoted $G(s)$, $M(s)$ and $N(s)$ with n_G , n_M , n_N .

For a simpler implementation of the tuning algorithm, the lowest values of the degrees are determined n_M , n_N from the conditions that the constructed system of algebraic equations satisfies the solvability condition, the physical feasibility of the control algorithm, and the robustness of the system:

$$\begin{aligned} n_G &\leq n_M + n_N + 1, \\ n_{A^-} + n_M &\leq n_{B^-} + n_N + r, \\ n_G &= n_{A^+} + n_N + r. \end{aligned} \quad (9)$$

The system (9) has a solution, if the degree n_G of the polynomial $G(s)$ satisfies the condition:

$$n_G - n_A \geq n_{A^+} + r + n_{B^-} - 1.$$

With the relation (6) the characteristic polynomial is constructed $G(s)$ of degree $n_G = 2 + k$ of the closed-loop system with the two dominant poles and k additional poles:

$$G(s) = (s + \alpha + j\omega)(s + \alpha - j\omega)(s + p_k)^k = (s^2 + 2\alpha s + a^2 + \omega^2)(s + p_k)^k, \quad (10)$$

where: k ($k = 1, 2, \dots$) is the number of additional negative real poles $-p_k$ added to satisfy the physical feasibility conditions of the controller and ensure the performance of the synthesized system.

In the expression (8) is substituted $G(s)$ with (10) and the equality is obtained:

$$B^+(s)M(s) + A^+(s)N(s)s^r = (s^2 + 2\alpha s + a^2 + \omega^2)(s + p_k)^k \quad (11)$$

from which the system of algebraic equations is constructed, by equating the coefficients of the same powers of the variable s on both sides of the equality, and the coefficients of the polynomials are calculated $M(s)$ and $N(s)$.

The transfer function of the control algorithm is determined from the relation:

$$H_R(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{Q(s)}{P(s)} = \frac{q_0 s^m + q_1 s^{m-1} + \dots + q_{m-1} s + q_{m_0}}{(p_0 s^n + p_1 s^{n-1} + \dots + p_{n-1} s + p_{n_p}) s^r}, \quad m_Q \leq n_P. \quad (12)$$

3. Applications and Computer Simulation

To highlight the advantages of the developed procedure for tuning the controller to the object model, (1)-(4) examples are analyzed after the polynomial method $n = 1, 2, 3, 4$.

Example 1. The object model is considered (1) of order $n = 1$ with known parameters $b_0 = 2$, $a_0 = 10$, $a_1 = 1$ described by the factorized transfer function:

$$H_1(s) = \frac{B(s)}{A(s)} = \frac{b_0}{a_0 s + a_1} = \frac{2}{10s + 1} = \frac{B^-(s)B^+(s)}{A^-(s)A^+(s)},$$

where $B^-(s) = b_0 = 2$, $B^+(s) = 1$, $A^-(s) = a_0 s + a_1 = 10s + 1$, $A^+(s) = 1$, with degree $n_{B^-} = n_{B^+} = 0$, $n_{A^-} = 1$, $n_{A^+} = 0$, $n_A = 1$.

It is required: For the damping ratio $\xi = 0.707$ and 0.8 , the settling time $t_s = 1$ s and the degree of astatism $r = 1$ to tune the controller to the object model (1) using the polynomial method.

Solution 1. The frequency is determined from (5):

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.707 \cdot 1} = 5.65777$$

and the dominant poles are calculated after (6):

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\alpha \pm j\omega = -0.707 \cdot 5.65777 \pm j5.65777 \cdot 0.707 = -4 \pm j4.$$

From the third condition of (9) to the degrees of $n_M = 0$, $n_N = 1$ and $r = 1$ the degree of the desired polynomial is determined: $n_G = n_{A^+} + n_N + 1 = 0 + 1 + 1 = 2$, and the unknown polynomials are described $M(s) = m_0$, $N(s) = n_0s + n_1$ and the desired characteristic polynomial is constructed:

$$B^+(s)M(s) + A^+(s)N(s)s = 1 \cdot m_0 + s(n_0s + n_1) = n_0s^2 + n_1s + m_0 = G(s). \quad (*)$$

The polynomial $G(s)$ of the closed-loop system is constructed with the two dominant poles with the degree $n = 2$:

$$G(s) = (s + 4 + j4)(s + 4 - j4) = s^2 + 8s + 32.$$

In the expression (*) is substituted $G(s)$ and the equality is obtained:

$$n_0s^2 + n_1s + m_0 = s^2 + 8s + 32,$$

from which, by equating the coefficients of the same powers of s from both the left-hand and right-hand sides, the values of the coefficients are obtained $n_0 = 1$, $n_1 = 8$, $m_0 = 32$.

The control algorithm is calculated using the relation (12) for the damping ratio $\xi = 0.707$ respectively:

$$H_{R11}(s) = \frac{Q(s)}{P(s)} = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s + a_1)}{b_0(n_0s^2 + n_1s)} = \frac{32(10s + 1)}{2(s^2 + 8s)} = \frac{320s + 32}{2s^2 + 16s}.$$

For the damping ratio $\xi = 0.8$, the settling time $t_s = 1$ s and the degree of astatism $r = 1$ to tune the controller to the object model (1) using the polynomial method.

Solution 2. The frequency is determined from (5):

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.8 \cdot 1} = 5$$

and the dominant poles are calculated after (6):

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\alpha \pm j\omega = -0.8 \cdot 5 \pm j5 \cdot 0.6 = -4 \pm j3.$$

From the third condition of (9) to the degrees of $n_M = 0$, $n_N = 1$ and $r = 1$ the degree of the desired polynomial is determined: $n_G = n_{A^+} + n_N + 1 = 0 + 1 + 1 = 2$, and the unknown polynomials are described $M(s) = m_0$, $N(s) = n_0s + n_1$ and the desired characteristic polynomial is constructed:

$$B^+(s)M(s) + A^+(s)N(s)s = 1 \cdot m_0 + 1 \cdot (n_0s + n_1)s = n_0s^2 + n_1s + m_0 = G(s). \quad (*)$$

The polynomial $G(s)$ of the closed-loop system is constructed with the two dominant poles with the degree $n = 2$:

$$G(s) = (s + 4 + j3)(s + 4 - j3) = s^2 + 8s + 25.$$

In the expression (*) is substituted $G(s)$ and the equality is obtained:

$$n_0s^2 + n_1s + m_0 = s^2 + 8s + 25,$$

from which, by equating the coefficients of the same powers of s from both the left-hand and right-hand sides, the values of the coefficients are obtained $n_0 = 1$, $n_1 = 8$, $m_0 = 25$.

The control algorithm is calculated using the relation (12) for the damping ratio $\xi = 0.8$ respectively:

$$H_{R12}(s) = \frac{Q(s)}{P(s)} = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s + a_1)}{b_0(n_0s^2 + n_1s)} = \frac{25(10s + 1)}{2(s^2 + 8s)} = \frac{250s + 25}{2s^2 + 16s}.$$

Example 2. The object model is considered (2) of order $n = 2$ with known parameters $b_0 = 2$, $a_0 = 50$, $a_1 = 15$, $a_2 = 1$ described by the factorized transfer function:

$$H_2(s) = \frac{b_0}{a_0s^2 + a_1s + a_2} = \frac{2}{50s^2 + 15s + 1} = \frac{B(s)}{A(s)} = \frac{B^-(s)B^+(s)}{A^-(s)A^+(s)}, \quad (2)$$

where $B^-(s) = b_0 = 2$, $B^+(s) = 1$, $A^-(s) = a_0s^2 + a_1s + a_2 = 50s^2 + 15s + 1$, $A^+(s) = 1$, with degree $n_{B^-} = n_{B^+} = 0$, $n_{A^-} = 2$, $n_{A^+} = 0$, $n_A = 2$.

It is required: For the damping ratio $\xi = 0.707$ and 0.8 , the settling time $t_s = 1s$ and the degree of astatism $r = 1$ to tune the controller to the object model (2) using the polynomial method.

Solution. The frequency is determined from (5):

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.707 \cdot 1} = 5.6577$$

and the dominant poles are calculated after (6):

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\alpha \pm j\omega = -0.707 \cdot 5.6577 \pm j5.6577 \cdot 0.707 = -4 \pm j4.$$

From the third condition of (9) to the degrees of $n_M = 0$, $n_N = 1$ and $r = 1$ the degree of the desired polynomial is determined: $n_G = n_{A^+} + n_N + r = 0 + 1 + 1 = 2$, and the unknown polynomials are described $M(s) = m_0$, $N(s) = n_0s + n_1$ and the desired characteristic polynomial is constructed:

$$B^+(s)M(s) + A^+(s)N(s)s = 1 \cdot m_0 + 1 \cdot (n_0s + n_1)s = n_0s^2 + n_1s + m_0 = G(s). \quad (*)$$

The polynomial $G(s)$ of the closed-loop system is constructed with the two dominant poles with the degree $n = 2$:

$$G(s) = (s + 4 + j4)(s + 4 - j4) = s^2 + 8s + 32.$$

In the expression (*) is substituted $G(s)$ and the equality is obtained:

$$n_0s^2 + n_1s + m_0 = s^2 + 8s + 32,$$

from which, by equating the coefficients of the same powers of s from both the left-hand and right-hand sides, the values of the coefficients are obtained $n_0 = 1$, $n_1 = 8$, $m_0 = 32$.

The control algorithm is calculated using the relation (12) for the damping ratio $\xi = 0.707$ and $\xi = 0.8$ (similar calculation), respectively:

$$\begin{aligned} H_{R21}(s) &= \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s^2 + a_1s + a_2)}{b_0(n_0s + n_1)s} = \frac{32(50s^2 + 15s + 1)}{2(s^2 + 8s)} \\ &= \frac{1600s^2 + 480s + 32}{2s^2 + 16s} \Big|_{\xi=0.707}, \end{aligned}$$

$$H_{R22}(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s^2 + a_1s + a_2)}{b_0(n_0s + n_1)s} = \frac{25(50s^2 + 15s + 1)}{2(s^2 + 8s)} \\ = \frac{1250s^2 + 375s + 25}{2s^2 + 16s} \Big|_{\xi=0.8}$$

Example 3. The object model is considered (3) of order $n = 3$ with known parameters $b_0 = 2$, $a_0 = 30$, $a_1 = 25$, $a_2 = 20$, $a_3 = 5$ described by the factorized transfer function:

$$H_3(s) = \frac{b_0}{a_0s^3 + a_1s^2 + a_2s + a_3} = \frac{2}{30s^3 + 25s^2 + 20s + 5} = \frac{B(s)}{A(s)} = \frac{B^-(s)B^+(s)}{A^-(s)A^+(s)}, \quad (3)$$

where $B^-(s) = b_0 = 2$, $B^+(s) = 1$, $A^-(s) = a_0s^3 + a_1s^2 + a_2s + a_3 = 30s^3 + 25s^2 + 20s + 5$, $A^+(s) = 1$, with degree $n_{B^-} = n_{B^+} = 0$, $n_{A^-} = 3$, $n_{A^+} = 0$, $n_A = 3$.

It is required: For the damping ratio $\xi = 0.707$ and 0.8 , the settling time $t_s = 1s$ and the degree of astatism $r = 1$ to tune the controller to the object model (3) using the polynomial method.

Solution. The frequency is determined from (5):

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.707 \cdot 1} = 5.6577$$

and the dominant poles are calculated after (6):

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\alpha \pm j\omega = -0.707 \cdot 5.6577 \pm j5.6577 \cdot 0.707 = -4 \pm j4.$$

From the third condition of (9) to the degrees of $n_M = 0$, $n_N = 2$ and $r = 1$ the degree of the desired polynomial is determined: $n_G = n_{A^+} + n_N + r = 0 + 2 + 1 = 3$, and the unknown polynomials are described $M(s) = m_0$, $N(s) = n_0s^2 + n_1s + n_2$ and the desired characteristic polynomial is constructed:

$$B^+(s)M(s) + A^+(s)N(s)s = 1 \cdot m_0 + s(n_0s^2 + n_1s + n_2) = n_0s^3 + n_1s^2 + n_2s + m_0 = G(s). \quad (*)$$

The polynomial $G(s)$ of the closed-loop system is constructed with the two dominant poles and additional negative real pole $p_3 = -30$ with the degree $n = 3$:

$$G(s) = (s^2 + 8s + 32)(s + 30) = s^3 + 38s^2 + 272s + 960.$$

In the expression (*) is substituted $G(s)$ and the equality is obtained:

$$n_0s^3 + n_1s^2 + n_2s + m_0 = s^3 + 38s^2 + 272s + 960,$$

from which, by equating the coefficients of the same powers of s from both the left-hand and right-hand sides, the values of the coefficients are obtained $n_0 = 1$, $n_1 = 38$, $n_2 = 272$, $m_0 = 960$.

The control algorithm is calculated using the relation (12) for the damping ratio $\xi = 0.707$ and $\xi = 0.8$ (similar calculation), respectively:

$$H_{R31}(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s^3 + a_1s^2 + a_2s + a_3)}{b_0(n_0s^2 + n_1s + n_2)s} = \frac{960(30s^3 + 25s^2 + 20s + 5)}{2(s^2 + 38s + 272)s} = \\ = \frac{28800s^3 + 24000s^2 + 19200s + 4800}{2s^3 + 76s^2 + 544s} \Big|_{\xi=0.707};$$

$$H_{R32}(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s^3 + a_1s^2 + a_2s + a_3)}{b_0(n_0s^2 + n_1s + n_2)s} = \frac{750(30s^3 + 25s^2 + 20s + 5)}{2(s^2 + 76s + 530)s} =$$

$$= \frac{22500s^3 + 18750s^2 + 15000s + 3750}{2s^3 + 76s^2 + 530s} \Big|_{\xi=0.8}$$

Example 4. The object model is considered (4) of order $n = 4$ with known parameters $b_0 = 2$, $a_0 = 30$, $a_1 = 24$, $a_2 = 20$, $a_3 = 15$, $a_4 = 5$ described by the factorized transfer function:

$$H_4(s) = \frac{b_0}{a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4} = \frac{2}{30s^4 + 24s^3 + 20s^2 + 15s + 5} = \frac{B(s)}{A(s)} = \frac{B^-(s)B^+(s)}{A^-(s)A^+(s)}, \quad (4)$$

where

$$B^-(s) = b_0 = 2,$$

$$B^+(s) = 1,$$

$A^-(s) = a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4 = 30s^4 + 24s^3 + 20s^2 + 15s + 5$, $A^+(s) = 1$, with degree $n_{B^-} = n_{B^+} = 0$, $n_{A^-} = 4$, $n_{A^+} = 0$, $n_A = 4$.

It is required: For the damping ratio $\xi = 0.707$ and 0.8 , the settling time $t_s = 1s$ and the degree of astatism $r = 1$ to tune the controller to the object model (1) using the polynomial method.

Solution. The frequency is determined from (5):

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.707 \cdot 1} = 5.6577$$

and the dominant poles are calculated after (6):

$$p_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\alpha \pm j\omega = -0.707 \cdot 5.6577 \pm j5.6577 \cdot 0.707 = -4 \pm j4.$$

From the third condition of (9) to the degrees of $n_M = 0$, $n_N = 3$ and $r = 1$ the degree of the desired polynomial is determined: $n_G = n_{A^+} + n_N + r = 0 + 3 + 1 = 4$, and the unknown polynomials are described $M(s) = m_0$, $N(s) = n_0s^3 + n_1s^2 + n_2s + n_3$ and the desired characteristic polynomial is constructed:

$$G(s) = (s^2 + 8s + 32)(s + 30)^2 = s^4 + 68s^3 + 1412s^2 + 9120s + 28800.$$

$$B^+(s)M(s) + A^+(s)N(s)s = 1 \cdot m_0 + s(n_0s^3 + n_1s^2 + n_2s + n_3) = n_0s^4 + n_1s^3 + n_2s^2 + n_3s + m_0 = G(s). \quad (*)$$

The polynomial $G(s)$ of the closed-loop system is constructed with the two dominant poles and additional negative real poles $p_{3,4} = -30$ with the degree $n = 4$:

In the expression (*) is substituted $G(s)$ and the equality is obtained:

$$n_0s^4 + n_1s^3 + n_2s^2 + n_3s + m_0 = s^4 + 68s^3 + 1412s^2 + 9120s + 28800,$$

from which, by equating the coefficients of the same powers of s from both the left-hand and right-hand sides, the values of the coefficients are obtained $n_0 = 1$, $n_1 = 68$, $n_2 = 1412$, $n_3 = 9120$, $m_0 = 28800$.

The control algorithm is calculated using the relation (12) for the damping ratio $\xi = 0.707$ and $\xi = 0.8$ (similar calculation), respectively:

$$H_{R42}(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4)}{b_0(n_0s^3 + n_1s^2 + n_2s + n_3)s} =$$

$$= \frac{28800(30s^4 + 24s^3 + 20s^2 + 15s + 5)}{2(s^3 + 68s^2 + 1412s + 9120)s} =$$

$$= \frac{864000s^4 + 691200s^3 + 576000s^2 + 432000s + 144000}{2s^4 + 136s^3 + 2824s^2 + 18240s} \Bigg|_{\xi=0.707};$$

$$H_{R42}(s) = \frac{A^-(s)M(s)}{B^-(s)N(s)s^r} = \frac{m_0(a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4)}{b_0(n_0s^3 + n_1s^2 + n_2s + n_3)s}$$

$$= \frac{22500(30s^4 + 24s^3 + 20s^2 + 15s + 5)}{2(s^3 + 68s^2 + 1405s + 8700)s} =$$

$$= \frac{675000s^4 + 540000s^3 + 450000s^2 + 337500s + 112500}{2s^4 + 136s^3 + 2810s^2 + 17400s} \Bigg|_{\xi=0.8}.$$

The automatic systems with models (1)-(4) and the synthesized controllers were simulated, and the step responses $h(t)$ are shown in Figure 2 ($\xi = 0.707$; $t_s = 1s$) and Figure 3 ($\xi = 0.8$; $t_s = 1s$), while the performances are given in Table 1 and Table 2.

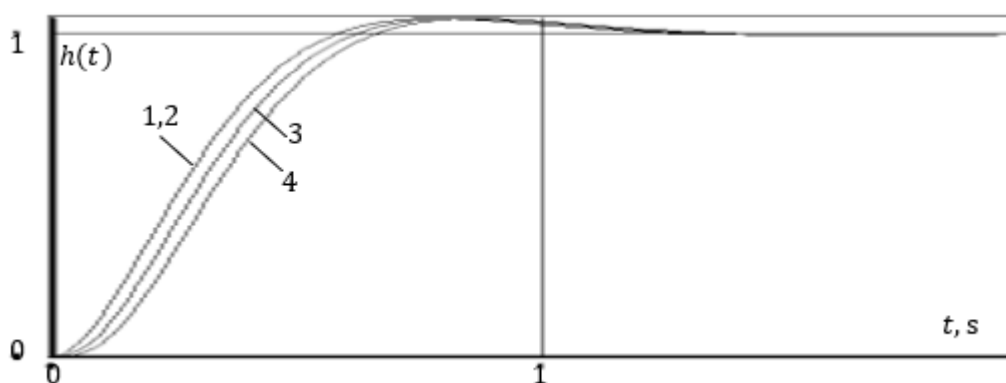


Figure 2. The step responses of the automatic systems ($\xi = 0.707$; $t_s = 1s$).

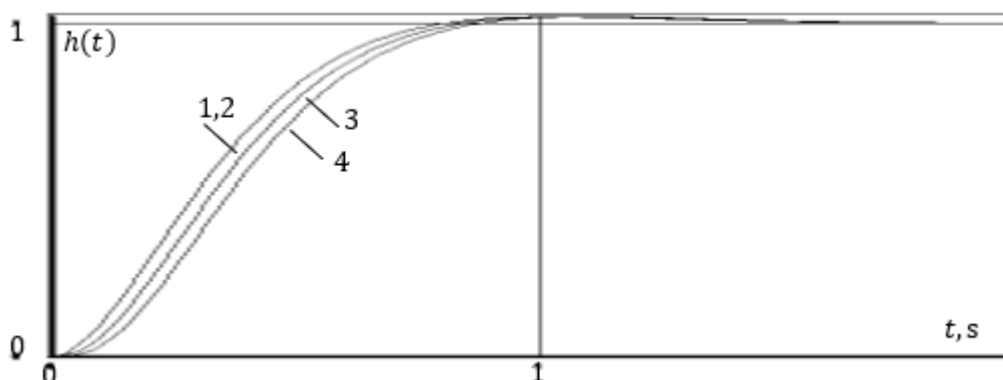


Figure 3. The step responses of the automatic systems ($\xi = 0.8$; $t_s = 1s$).

The numbering of the curves in Figures 2 and 3 corresponds to the degree of the object model in Table 1 and Table 2.

Table 1

The system's performance ($\xi = 0.707$; $t_s = 1s$)

Degree model	$\varepsilon, \%$	t_r, s	t_d, s	$d, \%$	t_s, s	n
1	5	0.51	0.78	4.86	0.51	1
2	5	0.51	0.78	4.86	0.51	1
3	5	0.55	0.82	4.86	0.55	1
4	5	0.58	0.86	4.86	0.58	1

Note: ε is the error of the steady-state system, t_r – the rise time, t_d – the time to reach the overshoot, d – the overshoot, t_s – the settling time, n – the number of oscillations during the transient regime.

Table 2

The system's performance ($\xi = 0.8$; $t_s = 1$ s)						
Degree model	ε , %	t_r , s	t_d , s	d , %	t_s , s	n
1	5	0.66	1.03	1.86	0.66	1
2	5	0.66	1.05	1.88	0.66	1
3	5	0.70	1.07	1.92	0.70	1
4	5	0.74	1.10	1.95	0.74	1

Note: ε is the error of the steady-state system, t_r – the rise time, t_d – the time to reach the overshoot, d – the overshoot, t_s – the settling time, n – the number of oscillations during the transient regime.

In the Table 3 calculates the degrees of stability of the automatic system for object models (1)-(4) and the synthesized regulator for the degree of damping $\xi = 0.707$ and $\xi = 0.8$.

Table 3

The degree of the model	Degree of stability	
	$\xi = 0.707, t_s = 1$ s	$\xi = 0.8, t_s = 1$ s
	1	-0.106
2	-0.066	-0.065
3	-0.234	-0.230
4	-0.302	-0.295

Note: ξ - the degree of damping, t_s - the settling time.

As the order of the control object model increases, the degree of stability increases and the system becomes more robust. And with the increase in the damping degree, the robustness of the system is reduced.

Next, the transfer functions of the open $H_d(s)$ and closed $H_0(s)$ automatic system are calculated for models (1)-(4) with the regulator synthesized at the values of the damping degree $\xi = 0.707$ and $\xi = 0.8$:

$$H_{d1}(s) = H_{R1}(s)H_1(s) = \frac{m_0(a_0s + a_1)}{b_0(n_0s + n_1)s} \frac{b_0}{a_0s + a_1} = \frac{m_0}{s(n_0s + n_1)},$$

$$H_{01}(s) = \frac{H_{R1}(s)H_1(s)}{1 + H_{R1}(s)H_1(s)} = \frac{m_0}{s(n_0s + n_1) + m_0} = \frac{m_0}{n_0s^2 + n_1s + m_0};$$

$$H_{d2}(s) = H_{R2}(s)H_2(s) = \frac{m_0(a_0s^2 + a_1s + a_2)}{b_0(n_0s + n_1)s} \frac{b_0}{(a_0s^2 + a_1s + a_2)} = \frac{m_0}{s(n_0s + n_1)},$$

$$H_{02}(s) = \frac{H_{R2}(s)H_2(s)}{1 + H_{R2}(s)H_2(s)} = \frac{m_0}{s(n_0s + n_1) + m_0} = \frac{m_0}{n_0s^2 + n_1s + m_0};$$

$$H_{d3}(s) = H_{R3}(s)H_3(s) = \frac{m_0(a_0s^3 + a_1s^2 + a_2s + a_3)}{b_0(n_0s^2 + n_1s + n_2)s} \frac{b_0}{(a_0s^3 + a_1s^2 + a_2s + a_3)} = \frac{m_0}{s(n_0s^2 + n_1s + n_2)},$$

$$H_{03}(s) = \frac{H_{R3}(s)H_3(s)}{1 + H_{R3}(s)H_3(s)} = \frac{m_0}{s(n_0s^2 + n_1s + n_2) + m_0} = \frac{m_0}{n_0s^3 + n_1s^2 + n_2s + m_0};$$

$$H_{d4}(s) = H_{R4}(s)H_4(s) = \frac{m_0(a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4)}{b_0(n_0s^3 + n_1s^2 + n_2s + n_3)s} \frac{b_0}{(a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4)} = \frac{m_0}{s(n_0s^3 + n_1s^2 + n_2s + n_3)},$$

$$H_{04}(s) = \frac{H_{R4}(s)H_4(s)}{1 + H_{R4}(s)H_4(s)} = \frac{m_0}{s(n_0s^3 + n_1s^2 + n_2s + n_3) + m_0} = \frac{m_0}{n_0s^4 + n_1s^3 + n_2s^2 + n_3s + m_0}.$$

From the analysis of the transfer functions of the synthesized closed automatic systems, it is found that the parameters of the systems are determined by the parameters of the polynomials $M(s)$ and $N(s)$.

The parameters of the polynomials $M(s)$ and $N(s)$ shall be calculated on the basis of the desired characteristic polynomial $G(s)$ of the synthesized system, constructed from the two dominant poles, determined by the performance imposed on the closed system, the degree of damping ξ and the adjustment time t_s and by the additional poles used to satisfy the conditions of physical feasibility of the synthesized regulator.

For the models of objects of order one and two, the transfer functions of the systems are equal and it follows that the performance of closed systems is also the same. The excess of poly-zeros of these systems is equal to two.

In the model of the object of order one the excess of poly-zeros is equal to two and is greater than the order of the object which is equal to one.

For the second, third and fourth order models the excess of poly-zeros of the transfer function of closed systems are equal to the order of the model $n = 2, 3, 4$.

4. Conclusions

Analyzing the results obtained when tuning the controller for models (1)-(4) with a damping ratio $\xi = 0.707$ and $\xi = 0.8$, and a settling time $t_s = 1s$ using the polynomial method, it is observed:

- The dynamic properties of the closed-loop synthesized system are determined by the properties of the desired polynomial – dependent on the poles imposed on the synthesized system.
- The step responses of the automatic systems are aperiodic.
- Within a 5% error margin, the synthesized systems have high performance.
- At the value of $\xi = 0.8$ and object models (1), (2) the rise time t_r is 1.29 times the rise time t_r at $\xi = 0.707$, and at the value of $\xi = 0.8$ and object models (3) and (4) the rise time t_r is 1.27 times the rise time t_r at $\xi = 0.707$.
- Automatic systems with first- and second-order object models have the same performance.
- As the degree of the object model increases, the performance gradually decreases.

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