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CONTROL-RELEVANT IDENTIFICATION OF THE FIRST-ORDER INERTIAL SYSTEMS WITH TIME-DELAY

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Abstract. This paper presents an approach for control-relevant identification of inertial systems with time delay. The experimental identification was conducted in closed-loop, and the control object is approximated by a first-order inertial transfer function with time delay. The coefficients were determined using simple analytical expressions based on values obtained from the undamped step response of the closed-loop system with a P controller. This study enhances the understanding of closed-loop experimental identification of inertial systems with time delay and offers effective strategies for synthesizing the PID control algorithm. The obtained results are validated through computer simulation using the MATLAB software package. The results confirm that the proposed identification method for first-order inertial systems provides accurate model estimation and that the control algorithm delivers good system performance.

Keywords: *experimental identification, synthesis of the PID control algorithm, models of object with inertia and time delay, closed-loop system identification.*

Rezumat. În această lucrare a fost propusă abordarea de identificarea experimentală a modelului matematic al procesului cu inerție de ordinul unu și timp mort, și sinteza algoritmului de conducere. Identificarea analitico-experimentală a fost realizată în buclă închisă, iar obiectul de conducere a fost propus de a fi aproximat cu funcția de transfer cu inerție de ordinul unu și timp mort, unde coeficienții au fost calculați pe baza expresiilor analitice simple conform valorilor extrase din răspunsul oscilant neamortizat al sistemului automat închis cu regulatorul P. Acest studiu contribuie la îmbunătățirea înțelegerii procesului de identificarea experimentală în buclă închisă a sistemelor inerțiale cu timp mort și oferă strategii eficiente pentru sinteza algoritmului de conducere PID. Rezultatele obținute au fost verificate prin simulare în pachetul de programe MATLAB. Rezultatele demonstrează că abordarea propusă pentru identificarea experimentală și sinteza algoritmului de conducere PID oferă rezultate bune în estimarea modelului matematic, iar algoritmul de conducere asigură performanțe ridicate ale sistemului automat.

Cuvinte cheie: *identificarea experimentală, sinteza algoritmului de conducere PID, modele de obiecte cu inerție și timp mort, identificarea modelului matematic în buclă închisă.*

1. Introduction

A precise mathematical model that approximates the dynamics of an industrial process is essential for the effective synthesis of control algorithms in industrial applications. Such a model can be obtained through various identification methods, conducted within either an open-loop or closed-loop structures of the automatic control system [1−4].

In an open-loop identification procedure, the system operates without feedback, allowing the input-output relationship to be directly observed. These methods are well-established and generally provide a reliable approximation of the process dynamics. The absence of feedback in this configuration simplifies the identification process, making it easier to apply standard techniques and obtain accurate models.

Conversely, closed-loop identification involves systems that operate with feedback, where the output is continuously fed back into the system to influence the input. This feedback loop complicates the identification process and standard open-loop identification methods are not directly applicable. In such scenarios, specialized closed-loop identification techniques must be implemented to accurately capture the process dynamics, accounting for the influence of feedback [5-7].

Nowadays, beside a big interest to the closed-loop identification, there is also significant attention being directed towards control-relevant identification [8]. Control-relevant identification is a specialized method for developing a mathematical model of a system with a focus in designing and tuning of the controller in real-time, ensuring that the identified models can tribute effectively to achieving desired control objectives. Unlike traditional identification methods, which aim for an accurate representation of the entire system, control-relevant identification prioritizes aspects that are most crucial for achieving specific control objectives. This approach emphasizes capturing the dynamics and characteristics that significantly impact control performance, such as stability, response time, and robustness [9-13].

This paper proposes to focus on system identification in a closed-loop configuration, aiming to estimate a first-order inertial system with time delay and to provide an algorithm for synthesizing the proportional-integral-derivative (PID) controller.

2. Closed-Loop System Identification of the First Order Inertial Process with Time Delay

The experimental identification algorithm is carried out within a closed-loop system using a P controller (Figure 1).

The P controller is described by [2]:

$$H_P(\mathrm{s}) = k_p, \tag{1}$$

where k_p - proportional tuning parameter of P controller.

The transfer function to be estimated is proposed as a first-order inertial model with time delay:

$$H(s) = \frac{ke^{-\tau s}}{Ts+1} = \frac{B(s)}{A(s)}, \tag{2}$$

where k is transfer coefficient, τ is time delay, T is time constant, $B(s) = ke^{-\tau s}$ and $A(s) = Ts + 1 = 0$.

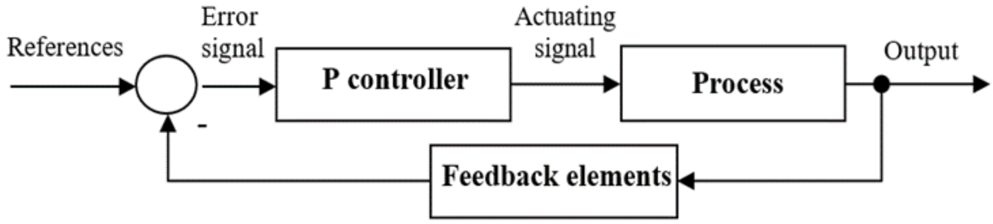

Figure 1. Block diagram of the automatic control system.

The system's transfer coefficient is calculated using the following relation [4]:

$$k = \lim_{t\to\infty} \frac{\Delta y}{\Delta u} = \lim_{t\to\infty} \frac{y_{st}-y_{initial}}{u-u_{initial}}, \tag{3}$$

where: y_{st} is the steady-state output value, $y_{initial}$ is the initial value of the output response, u – input signal, $u_{initial}$ is the initial value of the input signal.

From the transfer function (2), the time constant is proposed to be calculated based on k_p - the proportional gain of the P controller at the stability limit, and T_0 - the oscillation period. To obtain the undamped step response of the closed-loop system with a PID controller, following the Ziegler-Nichols method, the integral and derivative gains k_i and k_d are set to zero, and the proportional gain $k_p > 0$ is adjusted until the system reaches the undamped step response shown in Figure 2 [14].

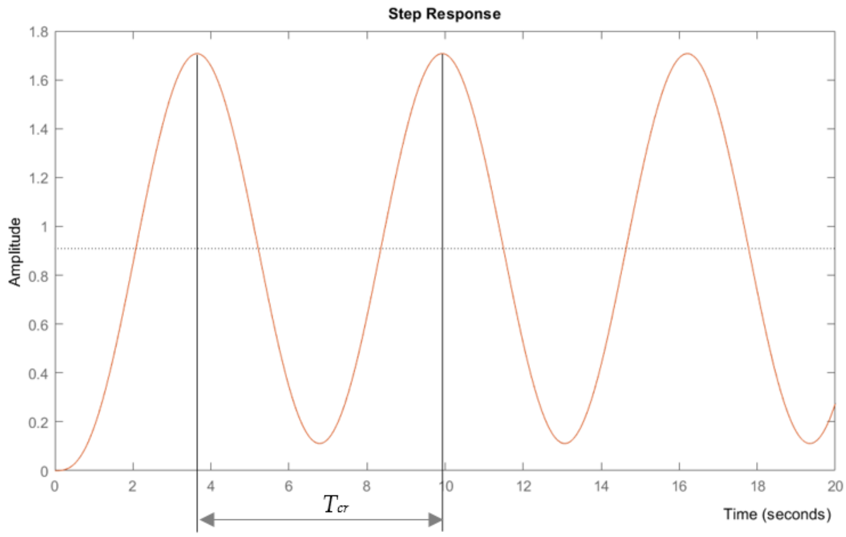

Figure 2. Undamped step response of the closed-loop system.

Next, the oscillation period T_{cr} is calculated from the undamped step response of the closed-loop system shown in Figure 2.

If the oscillation period is known, the natural frequency is calculated using the following expression [2]:

$$\omega_n = \frac{2\pi}{T_{cr}}. \tag{6}$$

The characteristic equation of the closed-loop system with a P controller is as follows:

$$A(s) = Ts + 1 + k \cdot k_{cr} \cdot e^{-\tau s} = 0. \tag{7}$$

The following substitution is proposed for the Laplace transform:

$$s = j\omega_n. \tag{8}$$

Based on the (8) the characteristic equation (7) will become:

$$A(j\omega_n) = jT\omega_n + 1 + k_{cr}ke^{-j\tau\omega_n} = (1 + k_{cr}k\cos(\tau\omega_n)) +$$
$$+ j(T\omega_n - k_{cr}k\sin(\tau\omega_n)) = P(\omega) + jQ(\omega) = 0,$$

where $P(\omega)$ is real part and $Q(\omega)$ is imaginary part.

Next, the imaginary part is set to zero, denoted as $Q(\omega) = 0$, and from this equation, the following expression for calculating the time constant is obtained:

$$T = \frac{k_{cr}k\sin(\tau\omega_n)}{\omega_n}. \tag{9}$$

3. Synthesis the PID Control Algorithm

The structure of the control system is shown in Figure 3 and consists of a controller with the transfer function $H_R(s)$ and a control object described by the transfer function (2).

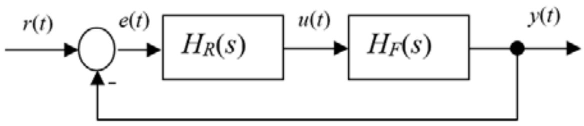

Figure 3. Structural block diagram of the automatic control system.

The PID controller is represented by the following transfer function [2]:

$$H_R(\mathrm{s}) = k_p + \frac{k_i}{s} + k_d s, \tag{10}$$

where: k_p, k_i and k_d are the tuning parameters of the PID controller.

According to the iterative maximum stability degree method, the analytical expressions for calculating the tuning parameters are as follows [15–16]:

$$\tau^2 T J^2 - (\tau^2 + 6\tau T)J + 3(\tau + 2T) = 0. \tag{11}$$

$$k_p = \frac{e^{-\tau J}}{k}(-\tau T J^2 + (\tau + 2T)J - 1) + 2k_d J = f_p(J), \tag{12}$$

$$k_i = \frac{e^{-\tau J}}{k}(-TJ^2 + J) - k_d J^2 + k_p J = f_i(J), \tag{13}$$

$$k_d = \frac{e^{-\tau J}}{2k}(-\tau^2 T J^2 + (\tau^2 + 4\tau T)J - 2(\tau + T)) = f_d(J), \tag{14}$$

where J is the stability degree of the system.,

According to relation (11), the expression for determining the stability degree can be obtained as follows:

$$J = \frac{\tau + 6T}{2\tau T}. \tag{15}$$

Next, based on the expression (15) and analytical expressions (13) and (14) there are obtained:

$$k_d = \frac{e^{-\tau J}}{2k} \cdot \frac{\tau^2 + 8T^2}{8T}, \tag{16}$$

$$k_i = \frac{e^{-\tau J}}{2k} \cdot \frac{\tau^2 - 36T^2}{4\tau^2 T} + \frac{k_p^2}{4k_d}, \tag{17}$$

From [17], it is known that

$$J = \frac{k_p}{2k_d}. \quad (18)$$

Based on (18) and expression (15), it can be obtained the relationship for calculation the value of proportional tuning coefficient:

$$k_p = 2 \cdot k_d \cdot J \quad (19)$$

4. Algorithm for Control-Relevant Identification

According to the proposed approach for closed-loop identification and PID control algorithm synthesis, the following algorithm for control-relevant identification is presented:

1. Collection of preliminary information.
2. Configuration of the feedback control system with a P controller.
3. Adjustment of the proportional gain $k_p > 0$ until the closed-loop system achieves an undamped step response.
4. Determination of the oscillation period T_{cr} and time delay τ from the undamped step response.
5. Calculation of the natural frequency using expression (6).
6. Estimation of the transfer function's parameters (2) based on expressions (3) and (9).
7. Tuning of the PID controller according to expressions (16)–(17) and (19).

5. Application and Computer Simulation

To verify the proposed algorithm of control-relevant identification, two examples of the control object model are examined.

Example 1. The control object is assumed to be described by the following transfer function:

$$H(s) = \frac{ke^{-\tau s}}{Ts+1} = \frac{0.5e^{-5s}}{100s+1} = \frac{B(s)}{A(s)}, \quad (20)$$

where: $B(s) = 0.5e^{-5s}$ and $A(s) = 100s + 1$.

Next, an automatic control system with a P controller was considered, and the parameter k_p was varied until the system response shown in Figure 4 was achieved.

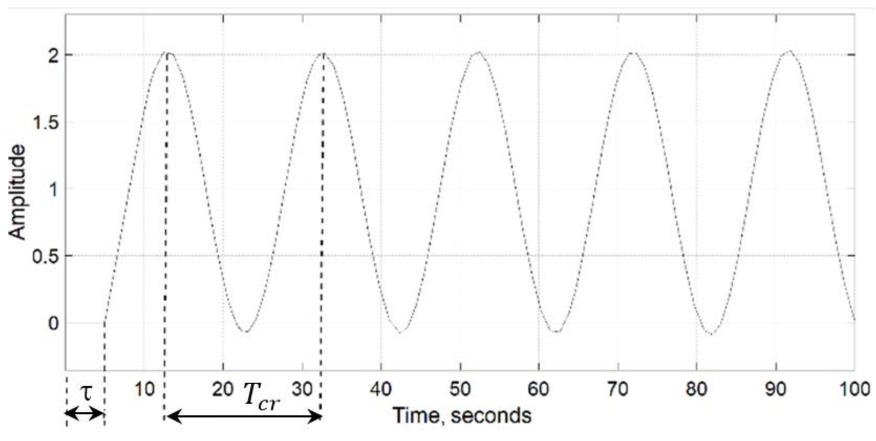

Figure 4. Undamped step response of the closed-loop system.

The following parameters were obtained: $k_{cr} = 64.8, T_{cr} = 19.659$ s.

According to the (6) expression, the value of natural frequency is $\omega_n = 0.3196$.

Knowing values of $T_{cr}, \omega_n, \tau, k$ and according to the expression (9), it is calculated the value of constant time T:

$$T = \frac{k_{cr} k \sin(\tau\omega_n)}{\omega_n} = 101.3392 \text{ s.} \tag{21}$$

Therefore, the resulting identified transfer function of the process is:

$$H_a(s) = \frac{ke^{-\tau s}}{Ts+1} = \frac{0.5e^{-5s}}{101.33s+1} = \frac{B(s)}{A(s)}, \tag{22}$$

where: $B(s) = 0.5e^{-5s}$ and $A(s) = 101.33s + 1$.

Figure 5 presents two system outputs: curve 1 shows the step response of the given transfer function (20), while curve 2 represents the step response of the identified model (22).

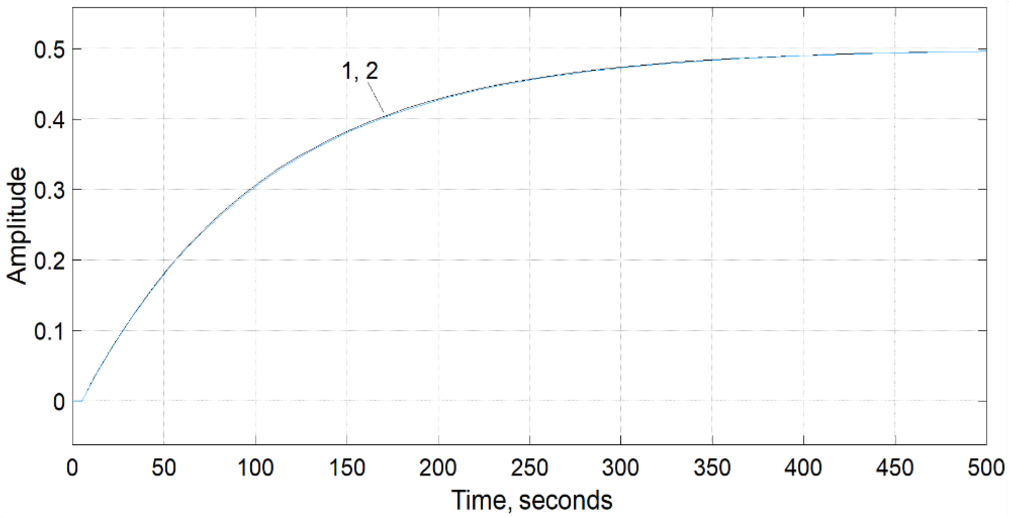

Figure 5. Comparation of the system step responses in the open loop.

Next, for the identified transfer function (22), the PID controller is tuned using expressions (16)–(17) and (19). Table 1 presents the calculated tuning parameter values and includes a comparison with the results obtained through parameter optimization in MATLAB.

Table 1

System Performance and Tuning Parameters of the PID Controller

No	Tuning Method	k_p	k_i, s^{-1}	k_d, s	t_s, s	t_r, s	σ, %
1	Proposed Method of Tuning	5.95	0.029	4.92	301	301	0
2	Parametrical Optimization	9.53	0.188	18.19	116.21	37.91	7.84

Note: k_p - proportional tuning parameter; k_i - integral tuning parameter; k_d - derivative tuning parameter; t_s - settling time; t_r - rise time; σ - overshoot.

Figure 6 shows the step responses of the automatic control system with a PID controller.

Example 2. Next, the control object is assumed to be represented by the following transfer function:

$$H(s) = \frac{e^{-\tau s}}{Ts+1} = \frac{e^{-20s}}{15s+1} = \frac{B(s)}{A(s)}, \tag{23}$$

where $B(s) = e^{-20s}$ and $A(s) = 15s + 1$.

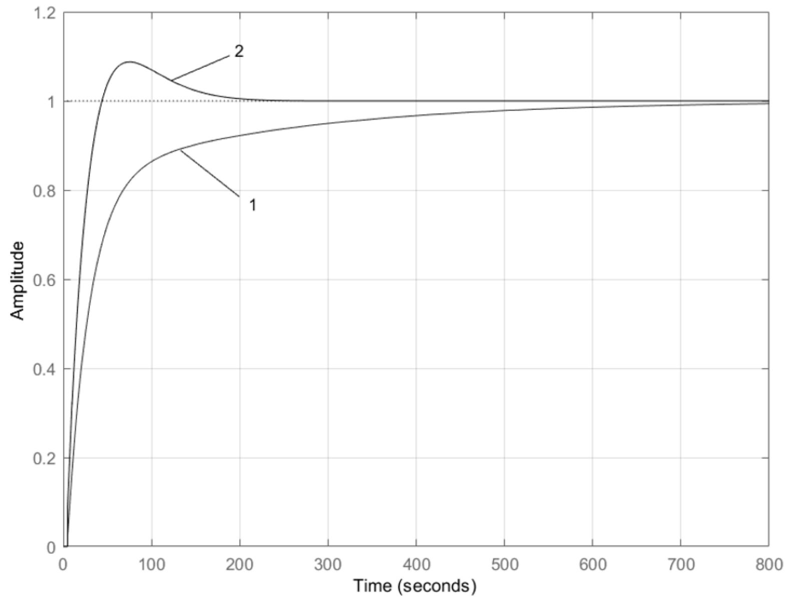

Figure 6. Step response of the automatic control system with PID controller tuned by: 1 – proposed method of tuning; 2 – parametrical optimization from MATLAB.

Next, an automatic control system with a P controller was considered, and the k_p parameter was adjusted until the system response shown in Figure 7 was obtained.

There are obtained the following critical system parameters:

$$k_{cr} = 1.9, T_{cr} = 58.541 \text{ s.}$$

According to the (6) expression, the value of natural frequency is $\omega_n = 0.1073$.

Knowing values of $T_{cr}, \omega_n, \tau, k$ and according to the expression (9), it is calculated the value of constant time T:

$$T = \frac{k_{cr} k \sin(\tau\omega_n)}{\omega_n} = 14.8583 \text{ s.} \tag{24}$$

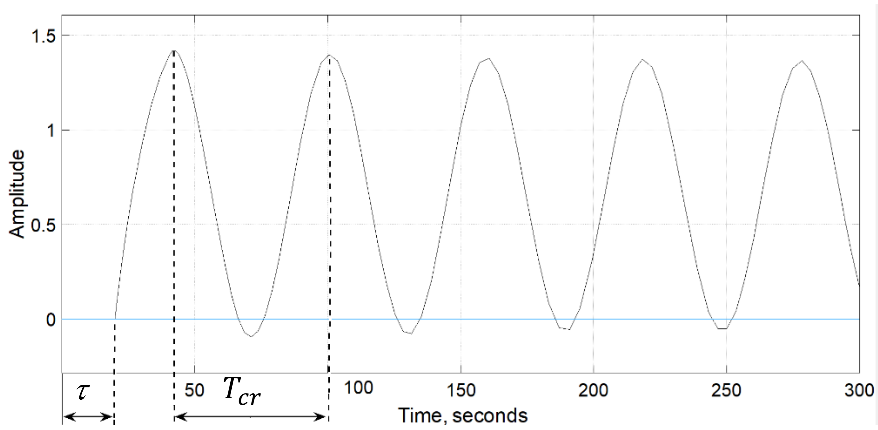

Figure 7. Undamped step response of the closed-loop system.

Thus, the identified transfer function of the process is:

$$H(s) = \frac{e^{-\tau s}}{Ts+1} = \frac{e^{-20s}}{14.8583s+1} = \frac{B(s)}{A(s)}. \tag{25}$$

Figure 8 presents two system outputs: curve 1 corresponds to the step response of the given transfer function (23), while curve 2 shows the step response of the identified model (25).

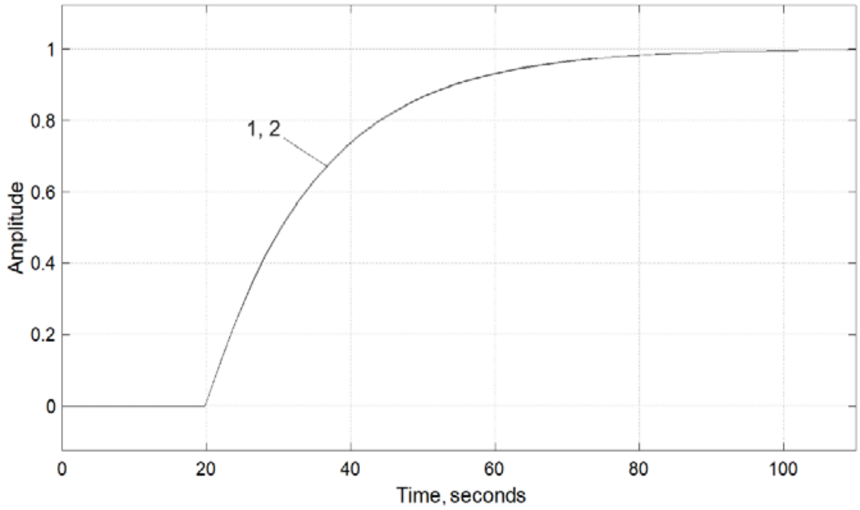

Figure 8. Comparation of the system step responses in the open loop.

Next, for the identified transfer function (25), the PID controller is tuned using expressions (16,17) and (19). Table 2 presents the calculated tuning parameter values, along with a comparison to the results obtained through parameter optimization in MATLAB.

Table 2

System Performance and Tuning Parameters of the PD Controller

No	Tuning Method	k_p	k_i, s^{-1}	k_d, s	t_s, s	t_r, s	$\sigma, \%$
1	Proposed Method of Tuning	0.085	0.0038	0.231	763.19	763.19	-
2	Parametrical Optimization	0.92	0.034	2.52	98.91	38.07	10.53

Note: k_p - proportional tuning parameter; k_i - integral tuning parameter; k_d - derivative tuning parameter; t_s - settling time; t_r - rise time; σ - overshoot.

Figure 9 presents the step responses of the automatic control system with a PID controller.

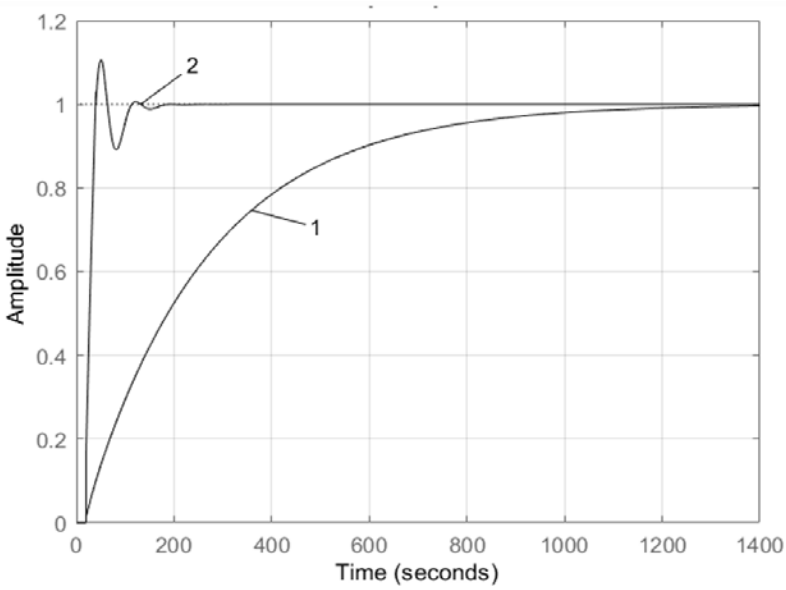

Figure 9. Step response of the automatic control system with PID controller tuned by: 1 – proposed method of tuning; 2 – parametrical optimization from MATLAB.

6. Conclusions

This paper proposed an algorithm for control-relevant identification of first-order inertial system with time delay. The algorithm begins with system identification conducted in closed-loop by achieving the system's undamped step response. Based on parameters extracted from this response, a mathematical model approximating the inertial system with time delay is calculated.

Next, simple analytical expressions are presented for calculating the PID controller tuning parameters. The proposed control-relevant identification algorithm was validated through computer simulation, with results compared against parameter optimization in MATLAB. This identification and PID tuning method can be implemented as an auto-tuning approach, providing the control system with high performance.

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Conflicts of Interest: The author declares no conflict of interest.

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